- ***Desarrollo de Taylor.***

Si una función posee un número infinito de derivadas, puede ser desarrollada en una serie de potencias:

$$f(x)=\sum_{n=0}^{\infty} a_n \left(x-x_0\right)^n \qquad \text{con} \qquad a_n=\frac{1}{n!}f^{(n)}(x_0)$$

$$f(x)=\sum_{n=0}^{\infty} \frac{1}{n!}f^{(n)}(x_0)\left(x-x_0\right)^n \ \rightarrow \ \text{función analítica}$$

Si la serie converge, los miembros tienden a cero cuando $n \rightarrow \infty$ y se puede cortar la serie después del n_p – *ésimo término.*

$$f(x)=\sum_{n=0}^{n_p} a_n \left(x-x_0\right)^n + R_{n_p}(x) \ , \quad \lim_{n_p\rightarrow\infty} R_{n_p}(x) \rightarrow 0$$

Para tres dimensiones:

Sea φ un campo escalar, al que queremos desarrollar en la vecindad del punto $\vec{r}$:
$\varphi\left(\vec{r}+\Delta\vec{r}\right)$.

Definimos F(t) como:

$$F(t)=\varphi\big(\underbrace{\vec{r}+t\cdot\Delta\vec{r}}_{\vec{g}(t)}\big)=\varphi\left(x_1+t\Delta x_1, x_2+t\Delta x_2, x_3+t\Delta x_3\right)=\varphi\left(g_1(t), g_2(t), g_3(t)\right)$$

$$\Rightarrow F(t)=\sum_{n=0}^{\infty}\frac{1}{n!}F^{(n)}(t_0)(t-t_0)^n \ , \ \text{si} \ \ t_0=0 \Rightarrow F(t)=\sum_{n=0}^{\infty}\frac{1}{n!}F^{(n)}(0)t^n \ \ ...(\Xi)$$

Usando la regla de la cadena:

$$\frac{dF(t)}{dt}=\sum_{i=1}^{3}\frac{\partial\varphi\big(\overbrace{\vec{r}+t\cdot\Delta\vec{r}}^{\vec{g}(t)}\big)}{\partial g_i}\frac{\partial g_i}{\partial t}=\sum_{i=1}^{3}\frac{\partial\varphi\left(\vec{g}(t)\right)}{\partial g_i}\Delta x_i \quad \text{, usando} \quad \frac{\partial g_i}{\partial x_i}=1$$

$$\Rightarrow \frac{dF(t)}{dt}=\sum_{i=1}^{3}\frac{\partial\varphi\left(\vec{g}(t)\right)}{\partial g_i}\frac{\partial g_i}{\partial x_i}\Delta x_i=\sum_{i=1}^{3}\frac{\partial\varphi\left(\vec{g}(t)\right)}{\partial x_i}\Delta x_i \ \ ... \ (\xi)$$

Ahora $F(0)=\varphi(\vec{r})$

$$F'(0)=\frac{dF(0)}{dt}=\sum_{i=1}^{3}\frac{\partial\varphi\left(\vec{r}\right)}{\partial x_i}\Delta x_i$$

De (ξ) tenemos que: $\displaystyle \frac{dF(t)}{dt}=\sum_{i=1}^{3}\frac{\partial\varphi\left(\vec{g}(t)\right)}{\partial x_i}\Delta x_i=\sum_{i=1}^{3}\frac{\partial F(t)}{\partial x_i}\Delta x_i$

$$\Rightarrow \frac{d}{dt}=\sum_{i=1}^{3}\frac{\partial}{\partial x_i}\Delta x_i=\sum_{i=1}^{3}\Delta x_i\frac{\partial}{\partial x_i}$$

Entonces:

$$F'(0) = \left(\sum_j \Delta x_j \frac{\partial}{\partial x_j} \right) \varphi(\vec{r})$$

$$F''(0) = \frac{d}{dt}\left(\frac{dF(t)}{dt} \right)_{t=0} = \sum_{j=1}^{3} \Delta x_j \frac{\partial}{\partial x_j} \left(\sum_{i=1}^{3} \frac{\partial \varphi\left(\vec{g}(t)\right)}{\partial x_i} \Delta x_i \right)_{t=0} = \sum_i \sum_j \Delta x_i \Delta x_j \frac{\partial^2}{\partial x_i \partial x_j} \varphi(\vec{r})$$

$\Rightarrow F''(0) = \left(\sum_j \Delta x_j \frac{\partial}{\partial x_j} \right)^2 \varphi(\vec{r})$ el superíndice 2 indica que debemos aplicar dos veces el operador

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$$F^{(n)}(0) = \left(\sum_j \Delta x_j \frac{\partial}{\partial x_j} \right)^n \varphi(\vec{r}) \ . \ . \ . \ (\xi\xi)$$

$\therefore de$ (Ξ) y ($\xi\xi$) tenemos que: $F(1) = \varphi\left(\vec{r} + \Delta\vec{r}\right) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\sum_j \Delta x_j \frac{\partial}{\partial x_j} \right)^n \varphi(\vec{r})$

$$\Rightarrow \varphi\left(\vec{r} + \Delta\vec{r}\right) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\Delta\vec{r} \bullet \nabla\right)^n \varphi(\vec{r})$$

$$\Rightarrow \varphi\left(\vec{r}_* + \Delta\vec{r}_*\right) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(\Delta\vec{r}_* \bullet \nabla^*\right)^n \varphi(\vec{r}_*) \qquad . \ . \ . \ (*)$$