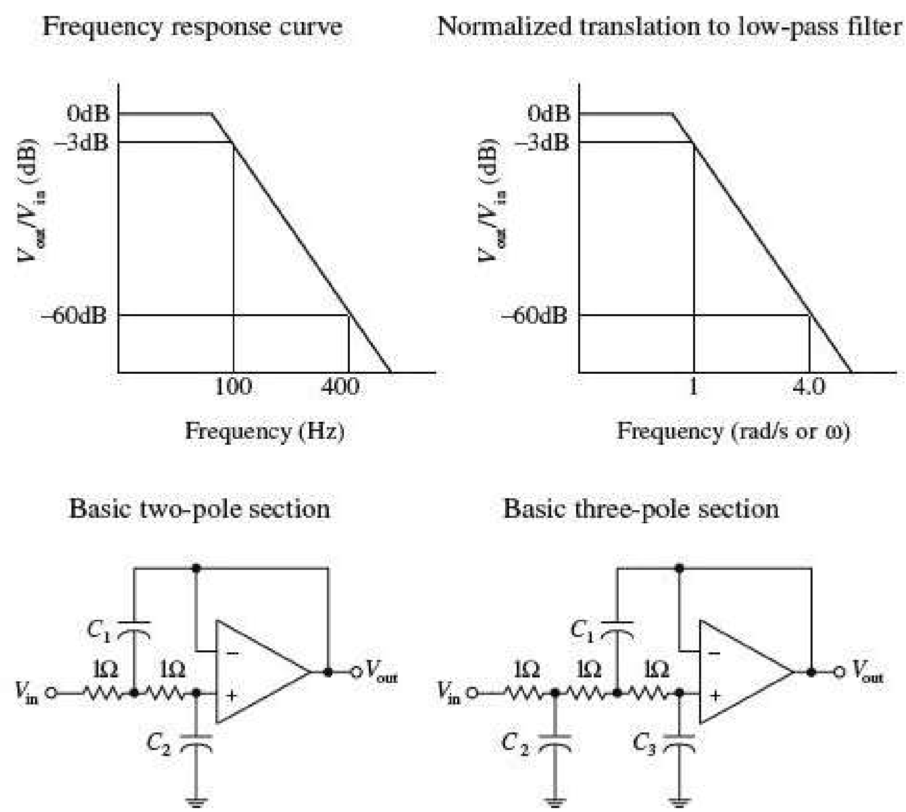
FIGURE 9.13

9.8 Active Filter Design

This section covers some basic Butterworth active filter designs. I already discussed the pros and cons of active filter design earlier in this chapter. Here I will focus on the actual design techniques used to make unity-gain active filters. To begin, let's design a low-pass filter.

9.8.1 Active Low-Pass Filter Example



Suppose that you wish to design an active low-pass filter that has a 3-dB point at 100 Hz and at least 60 dB worth of attenuation at 400 Hz—which we'll call the *stop frequency* f_s.

The first step in designing the filter is to normalize low-pass requirements. The steepness factor is

$$A_s = \frac{f_s}{f_{3dB}} = \frac{400\text{ Hz}}{100\text{ Hz}} = 4$$

This means that the normalized position of f_s is set to 4 rad/s. See the graphs in Fig. 9.14. Next, use the Butterworth low-pass filter response curves in Fig. 9.6 to determine the order of filter you need. In this case, the $n = 5$ curve provides over −60 dB at 4 rad/s. In other words, you need a fifth-order filter.

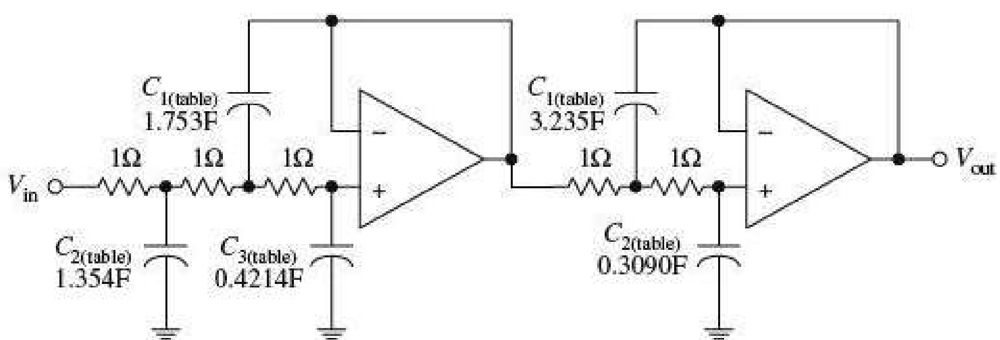
Now, unlike passive filters design, active filter design requires the use of a different set of basic normalized filter networks and a different table to provide the components of the networks. The active filter networks are shown in Fig. 9.14—there are two of them. The one to the left is called a *two-pole section*, while the one on the right is called a *three-pole section*. To design a Butterworth low-pass normalized filter of a given order, use Table 9.2. (Filter handbooks provide Chebyshev and Bessel tables as well.) In this example, a five-pole filter is needed, so according to the table, two sections are required—a three-pole and a two-pole section. These sections are cascaded together, and the component values listed in Table 9.2 are placed next to the corresponding components within the cascaded network. The resulting normalized low-pass filter is shown in Fig. 9.15.

FIGURE 9.14

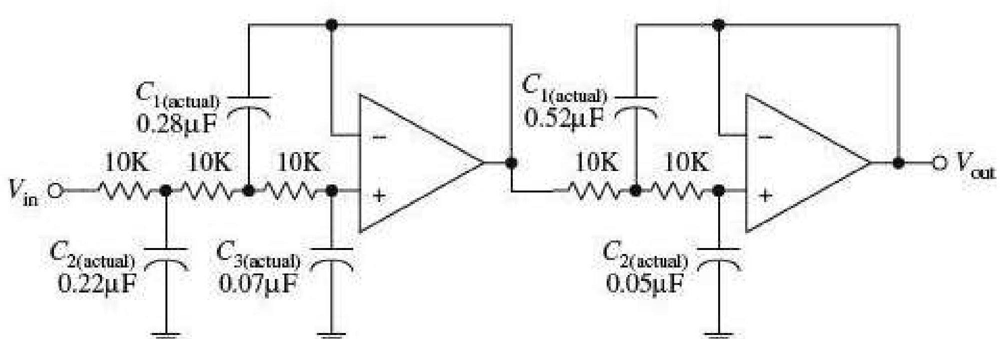
TABLE 9.2 Butterworth Normalized Active Low-Pass Filter Values

ORDER n	NUMBER OF SECTIONS	SECTIONS	C_1	C_2	C_3
2	1	2-pole	1.414	0.7071	
3	1	3-pole	3.546	1.392	0.2024
4	2	2-pole	1.082	0.9241	
		2-pole	2.613	0.3825	
5	2	3-pole	1.753	1.354	0.4214
		2-pole	3.235	0.3090	
6		2-pole	1.035	0.9660	
	3	2-pole	1.414	0.7071	
		2-pole	3.863	0.2588	
7		3-pole	1.531	1.336	0.4885
	3	2-pole	1.604	0.6235	
		2-pole	4.493	0.2225	
8		2-pole	1.020	0.9809	
	4	2-pole	1.202	0.8313	
		2-pole	2.000	0.5557	
		2-pole	5.758	0.1950	

Normalized low-pass filter



Final low-pass filter



The normalized filter will provide the correct response, but the component values are impractical—they're too big. In order to bring these values down to size, the circuit must be frequency and impedance scaled. To frequency scale, simply divide the capacitor values by $2\pi f_{3dB}$ (you need not frequency scale the resistors—they aren't reactive). In terms of impedance scaling, you do not have to deal with source/load impedance matching. Instead, simply multiply the normalized filter circuit's resistors by a factor of *Z* and divide the capacitors by the same factor. The value of *Z* is chosen to scale the normalized filter components to more practical values. A typical value for *Z* is 10,000 Ω. In summary, the final scaling rules are expressed as follows:

$$C_{(\text{actual})} = \frac{C_{(\text{table})}}{Z \cdot 2\pi f_{3\text{dB}}}$$

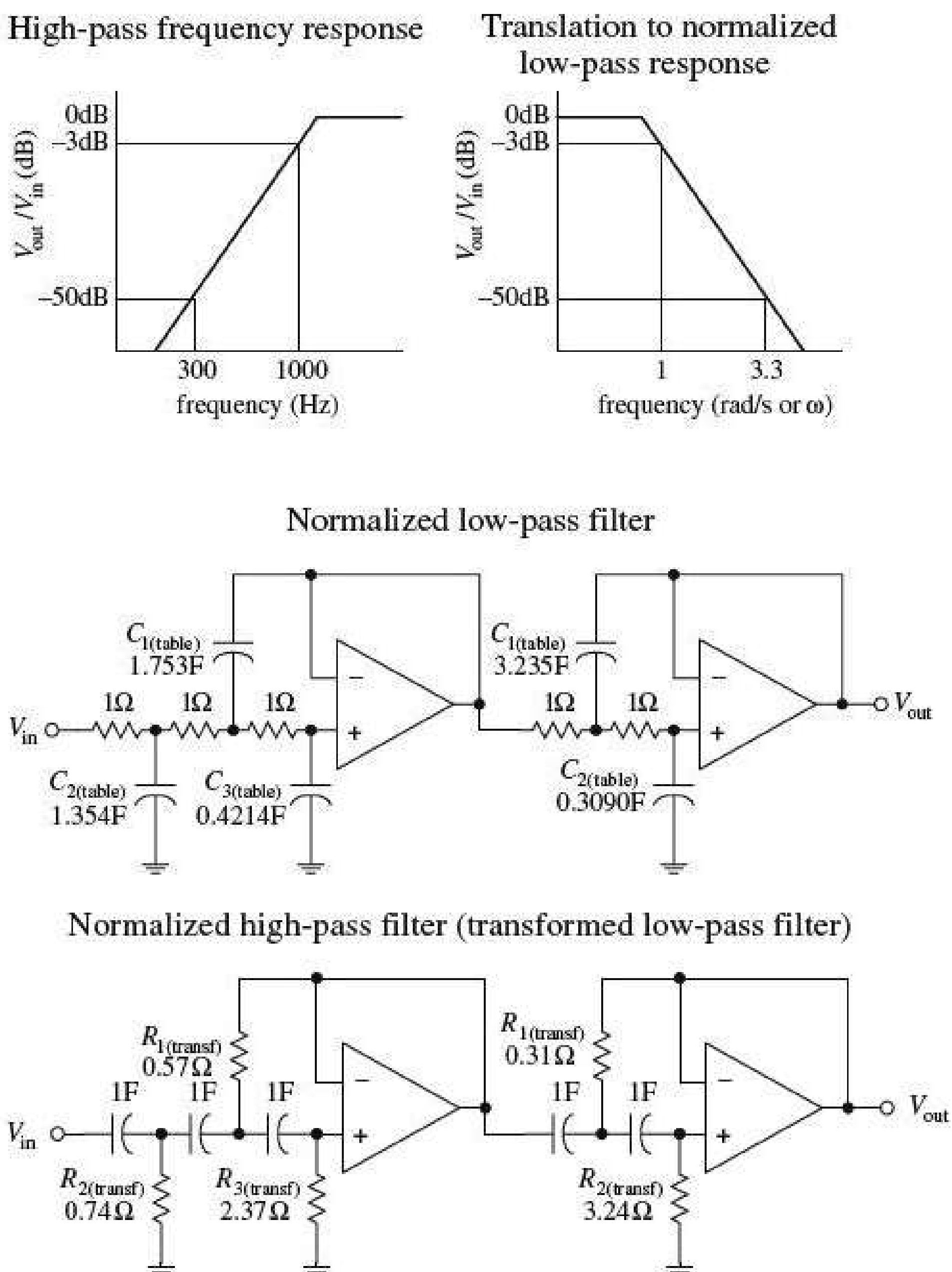
$$R_{(\text{actual})} = ZR_{(\text{table})}$$

Taking Z to be 10,000 Ω, you get the final low-pass filter circuit shown at the bottom of the figure.

FIGURE 9.15

9.8.2 Active High-Pass Filter Example

The approach used to design active high-pass filters is similar to the approach used to design passive high-pass filters. Take a normalized low-pass filter, transform it into a high-pass circuit, and then frequency and impedance scale it. For example, suppose that you want to design a high-pass filter with a −3-dB frequency of 1000 Hz and 50 dB worth of attenuation at 300 Hz. What do you do?



The first step is to convert the high-pass response into a normalized low-pass response, as shown in the figure. The steepness factor for the low-pass equivalent response is given by

$$A_s = \frac{f_{3\mathrm{dB}}}{f_s} = \frac{1000\ \mathrm{Hz}}{300\ \mathrm{Hz}} = 3.3$$

This means that the stop frequency is set to 3.3 rad/s on the normalized graph. The Butterworth response curve shown in Fig. 9.6 tells you that a fifth-order (n = 5) filter will provide the needed attenuation response. Like the last example, a cascaded three-pole/two-pole normalized low-pass filter is required. This filter is shown in Fig. 9.16.

Next, the normalized low-pass filter must be converted into a normalized high-pass filter. To make the conversion, exchange resistors for capacitors that have values of 1/*R* F, and exchange capacitors with resistors that have values of 1/*C* Ω. The second circuit in Fig. 9.16 shows the transformation.

Like the last example problem, to construct the final circuit, the normalized high-pass filter's component values must be frequency and impedance scaled:

$$C_{(\mathrm{actual})} \frac{C_{(\mathrm{transf})}}{Z \cdot 2\pi f_{3\mathrm{dB}}}$$

$$R_{(\mathrm{actual})} = ZR_{(\mathrm{transf})}$$

Again, let Z = 10,000 Ω. The final circuit is shown in Fig. 9.17.

FIGURE 9.16

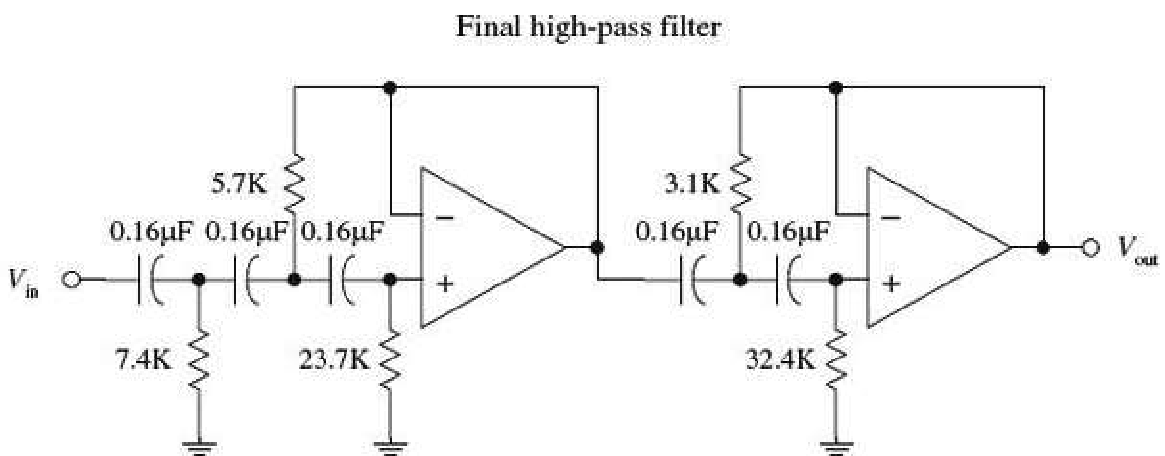


FIGURE 9.17

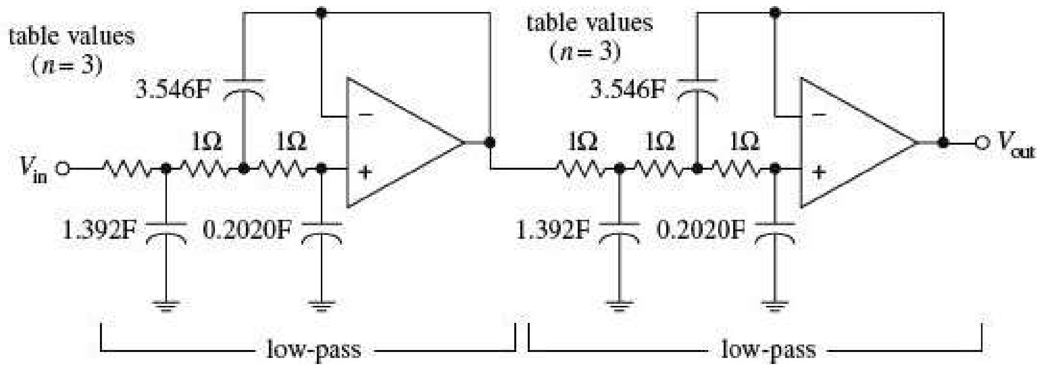
9.8.3 Active Bandpass Filters

To design an active bandpass filter, it is necessary to determine if a wide-band or narrow-band type is needed. If the upper 3-dB frequency divided by the lower 3-dB frequency is greater than 1.5, the bandpass filter is a wide-band type; below 1.5, it is a narrow-band type. To design a wide-band bandpass filter, simply cascade a high-pass and low-pass active filter together. To design a narrow-band bandpass filter, you have to use some special tricks.

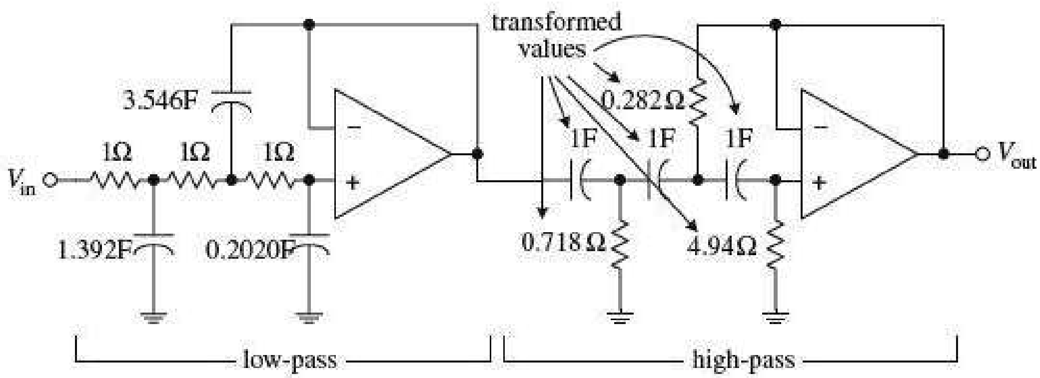
Wide-Band Example

Suppose that you want to design a bandpass filter that has Ω3-dB points at f_1 = 1000 Hz and f_2 = 3000 Hz and at least −30 dB at 300 and 10,000 Hz. What do you do?

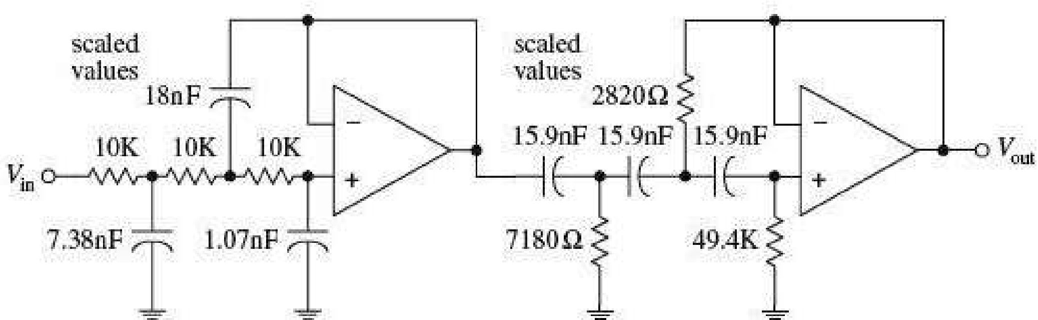
Normalized low-pass/low-pass initial setup



Normalized and transformed bandpass filter



Final bandpass filter



First, confirm that this is a wide-band situation:

$$\frac{f_2}{f_1} = \frac{3000\text{ Hz}}{1000\text{ Hz}} = 3$$

Yes it is—it is greater than 1.5. This means that you simply have to cascade a low-pass and high-pass filter together. Next, the response requirements for the bandpass filter are broken down into low-pass and high-pass requirements:

Low-pass:	–3 dB at 3000 Hz
	–30 dB at 10,000 Hz
High-pass:	–3 dB at 1000 Hz
	–30 dB at 300 Hz

The steepness factor for the low-pass filter is

$$A_s = \frac{f_s}{f_{3\text{dB}}} = \frac{10{,}000\text{ Hz}}{3000\text{ Hz}} = 3.3$$

while the steepness factor for the high-pass filter is

$$A_s = \frac{f_{3\text{dB}}}{f_s} = \frac{1000 \text{ Hz}}{300 \text{ Hz}} = 3.3$$

This means that the normalized stop frequencies for both filters will be 3.3 rad/s. Next, use the response curves in Fig. 9.6 to determine the needed filter orders—*n* = 3 provides over −30 dB at 3.3 rad/s. To create the cascaded, normalized low-pass/high-pass filter, follow the steps in the last two examples. The upper two circuits in the figure show the steps involved in this process. To construct the final bandpass filter, the normalized bandpass filter must be frequency and impedance scaled.

Low-pass section:

$$C_{(\text{actual})} = \frac{C_{\text{table}}}{Z \cdot 2\pi f_{3\text{dB}}} = \frac{C_{\text{table}}}{Z \cdot 2\pi(3000 \text{ Hz})}$$

High-pass section:

$$C_{(\text{actual})} = \frac{C_{\text{table}}}{Z \cdot 2\pi f_{2\text{dB}}} = \frac{C_{\text{table}}}{Z \cdot 2\pi(1000 \text{ Hz})}$$

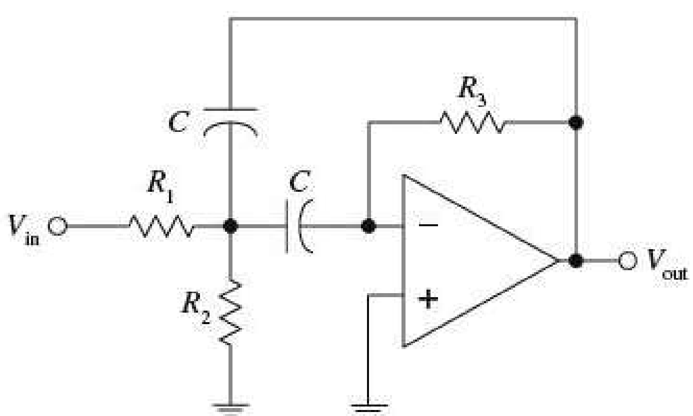
Choose *Z* = 10,000 Ω. to provide convenient scaling of the components. In the normalized circuit, resistors are multiplied by a factor of *Z*. The final bandpass filter is shown at the bottom of the figure.

FIGURE 9.18

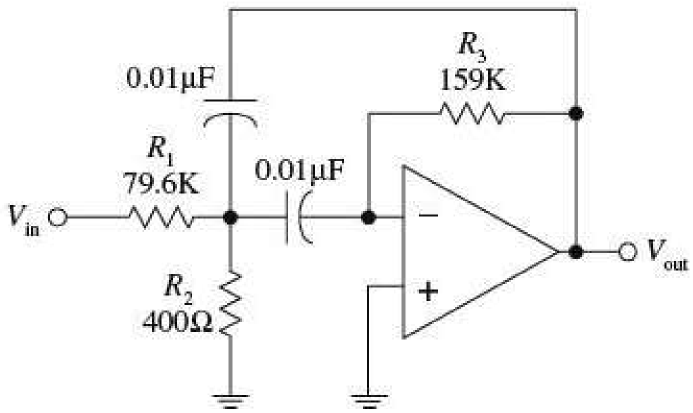
Narrow-Band Example

Suppose that you want to design a bandpass filter that has a center frequency f_0 = 2000 Hz and a −3-dB bandwidth $\Delta f_{BW} = f_2 - f_1$ = 40 Hz. How do you design the filter? Since f_2/f_1 = 2040 Hz/1960 Hz = 1.04, it is not possible to used the low-pass/high-pass cascading technique you used in the wide-band example. Instead, you must use a different approach. One simple approach is shown below.

Narrow-band filter circuit



Final filter circuit



In this example, simply use the circuit in [Fig. 9.19] and some important equations that follow. No detailed discussion will ensue.
First, find the quality factor for the desired response:

$$Q = \frac{f_0}{f_2 - f_1} = \frac{2000\ \text{Hz}}{40\ \text{Hz}} = 50$$

Next, use the following design equations:

$$R_1 = \frac{Q}{2\pi f_0 C} \qquad R_2 = \frac{R_1}{2Q^2 - 1} \qquad R_3 = 2R_1$$

Picking a convenient value for *C*—which we'll set to 0.01 µF—the resistors' values become

$$R_1 = \frac{50}{2\pi(2000\ \text{Hz})(0.01\ \mu\text{F})} = 79.6\ \text{k}\Omega$$

$$R_2 = \frac{79.6\ \text{k}\Omega}{2(50)^2 - 1} = 400\ \Omega$$

$$R_3 = 2(79.6\ \text{k}\Omega) = 159\ \text{k}\Omega$$

The final circuit is shown at the bottom of the figure. R_2 can be replaced with a variable resistor to allow for tuning.

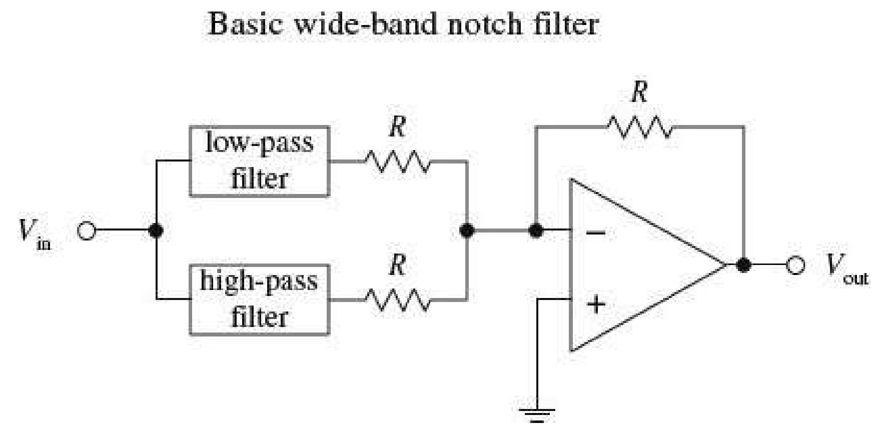
FIGURE 9.19

9.8.4 Active Notch Filters

Active notch filters come in narrow- and wide-band types. If the upper −3-dB frequency divided by the lower −3-dB frequency is greater than 1.5, the filter is called a *wide-band notch filter*—less than 1.5, the filter is called a *narrow-band notch filter*.

Wide-Band Notch Filter Example

To design a wide-band notch filter, simply combine a low-pass and high-pass filter together as shown in Fig. 9.20.



For example, if you need a notch filter to have −3-dB points at 500 and 5000 Hz and at least −15 dB at 1000 and 2500 Hz, simply cascade a low-pass filter with a response of

3 dB at 500 Hz
15 dB at 1000 Hz

with a high-pass filter with a response of

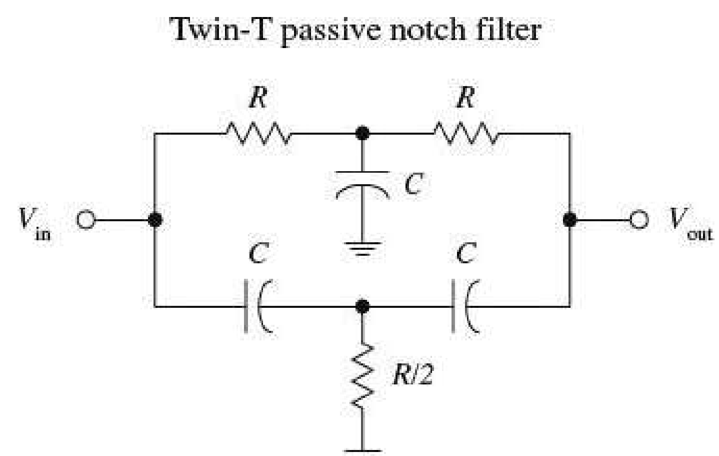
3 dB at 5000 Hz
15 dB at 2500 Hz

After that, go through the same low-pass and high-pass design procedures covered earlier. Once these filters are constructed, combine them as shown in the circuit in Fig. 9.20. In this circuit, *R* = 10 k is typically used.

FIGURE 9.20

Narrow-Band Notch Filters Example

To design a narrow-band notch filter ($f_2/f_1 < 1.5$), an *RC* network called the *twin-T* (see Fig. 9.21) is frequently used. A deep null can be obtained at a particular frequency with this circuit, but the circuit's *Q* is only ¼. (Recall that the *Q* for a notch filter is given as the center or null frequency divided by the −3-dB bandwidth.) To increase the *Q*, use the active notch filter shown in Fig. 9.22.

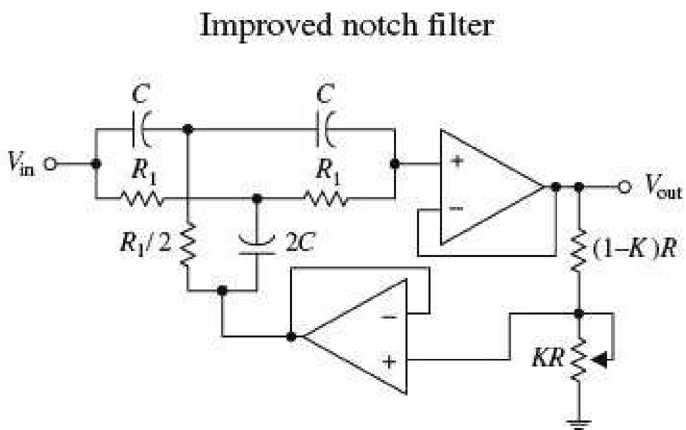


Like the narrow-bandpass example, let's simply go through the mechanics of how to pick the component values of the active notch filter. Here's an example.

Suppose that you want to make a "notch" at f_0 = 2000 Hz and desire a −3-dB bandwidth of Δf_{BW} = 100 Hz. To get this desired response, do the following. First determine the *Q:*

$$Q = \frac{\text{"notch" frequency}}{-3\text{-dB bandwidth}} = \frac{f_0}{\Delta f_{BW}} = \frac{2000\text{ Hz}}{100\text{ Hz}} = 20$$

FIGURE 9.21



The components of the active filter are found by using

$$R_1 = \frac{1}{2\pi f_0 C} \qquad \text{and} \qquad K = \frac{4Q-1}{4Q}$$

Now arbitrarily choose *R* and *C*; say, let *R* = 10 k and *C* = 0.01 μF. Next, solve for R_1 and *K*:

$$R_1 = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi(2000\ \text{Hz})(0.01\ \mu\text{F})} = 7961\ \Omega$$

$$K = \frac{4Q-1}{4Q} = \frac{4(20)-1}{4(20)} = 0.9875$$

Substitute these values into the circuit in Fig. 9.22. Notice the variable potentiometer—it is used to fine-tune the circuit.

FIGURE 9.22

9.9 Integrated Filter Circuits

A number of filter ICs are available on the market today. Two of the major categories of integrated filter circuits include the state-variable and switched-capacitor filter ICs. Both these filter ICs can be programmed to implement all the second-order functions described in the preceding sections. To design higher-order filters, a number of these ICs can be cascaded together. Typically, all that's needed to program these filter ICs is a few resistors. Using IC filters allows for great versatility, somewhat simplified design, good precision, and limited design costs. Also, in most applications, frequency and selectivity factors can be adjusted independently.

An example of a state-variable filter IC is the AF100 made by National Semiconductor. This IC can provide low-pass, high-pass, bandpass, and notch filtering capabilities (see Fig. 9.23). Unlike the preceding filters covered in this chapter, the state-variable filter also can provide voltage gain.

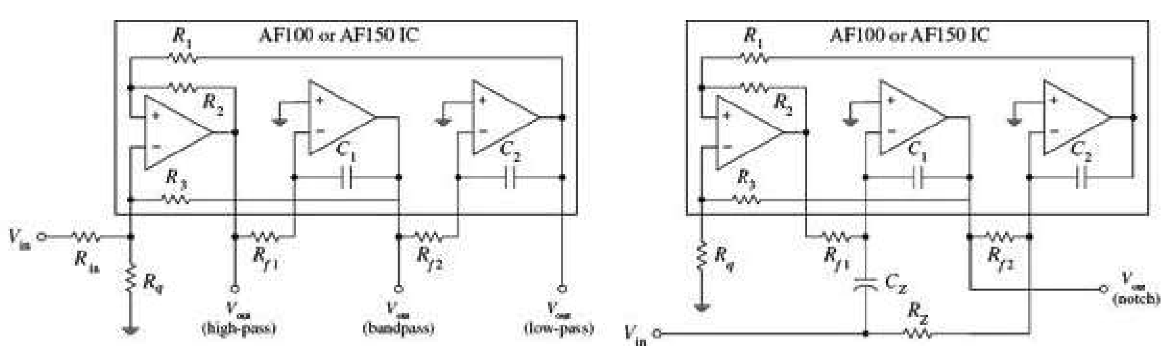


FIGURE 9.23