

REPRESENTATION AND QUALITY OF THE ANSWERS, IDENTIFIED BY DIGRAPHS, OBTAINED IN CONDITIONS OF COLLABORATIVE SCHOOL LEARNING

Patricia E. Balderas Cañas empatbal@unam.mx balderas.patricia@gmail.com

June 23-26, 2019

## CONTENT

o Abstract

- Background
- Research question
- Methodological aspects
- Results and discussion
o Conclusions



## ABSTRACT

- Here the main objective is to identify the quality of the answers obtained in conditions of collaborative school learning by high school students.
- The students' written answers were the source to construct digraphs that allowed identifying the representation (Balderas, 2018) and the quality of the answers when they are compared with a criterion (the response of the teacher).
- The analysis of the responses of 19 high school students, who worked collaboratively guided by a learning material and the use of advanced calculators, to the item:


## ABSTRACT

- "Evaluate the change that occurs in the independent variable when you move from the position occupied by point $\mathrm{A}(-3.0)$ to the position of point $B(-2,-3)$, in the graph of the function $y=x^{2}+2 x-3$. Explain your answer ".
- The quality of the discursive response of 19 participants was good because 9 of them had an index higher than 0.543 , although they were not located in the conceptual framework, due to the low conceptual structure and low quality in correspondence with the concepts and relationships contained in the criterion.



## BACKGROUND

- The concepts and relations into a text provide elements to study the quality of the answers gotten in conditions of school learning.
- The learning concept behind this analysis is based on the building and organization of categories in the classroom, as a cause and effect of several processes, between others, didactic process (Campos and Gaspar, 1996)


## LEARNING AND STUDENTS' ROLE

- For MAP, learning is a process of construction of the cognitive apparatus that includes the acquisition of knowledge, development of skills and interaction between the first two.
- Students' role is to be a teacher responsible for their own learning (Light and Cox, 2001).


## RESEARCH QUESTION

- How does the learner acquire mathematical concepts and how do they organize them to produce acceptable responses in the school environment?

MeThOdOLOGICAL ASPECTS

## METHODOLOGICAL ASPECTS

- Collection of participants' responses to the activities proposed in an instrument (naturalistic study, case study format, Lincoln and Guba, 1995)
- Construction of propositional maps by the Propositional Analysis Model, MAP (Acronym in Spanish, Campos and Gaspar, 1995)
- Preparation of digraphs (Harary, 1965; Balderas 1998 and 2018)
- Determination of the overall quality of each response in terms of MAP (comparison of response with criterion)
- Validation of answers through interviews
- Elaboration of conclusions


## PHYSICAL CONDITIONS FOR RESPONSE COLLECTION



## INTERACTION IN SCHOOL LEARNING (BLOOME, 1992)

- Solution of activities in small groups
- How do students organize their knowledge?


INTERACTION OF PARTICIPANTS IN SMALL GROUPS USE OF BODY AND KINETIC LANGUAGE


## ITEM

- Evaluate the change that occurs in the independent variable when going from the position occupied by point $\mathrm{A}(-3.0)$ to the position of point $\mathrm{B}(-2,-3)$, in the graph of the function

$$
y=x^{2}+2 x-3
$$

- Explain your answer.


## InTERPRETIVE FRAMEWORK

- Density $(\boldsymbol{d})$ is an indicator of the coherence of discourse and is calculated by the ratio between the number of concepts and relationships contained in the student's response.
- The organization of knowledge embodied in each map allowed to identify the correspondence between the concepts of the participants' answers and the criterion (cc).
- As well as, the correspondence of relationships involved in the conceptual correspondence, of the participant and the

$$
d=\frac{C}{R}
$$

$$
c c=\frac{C_{S T}}{C_{T}}
$$ criterion (cr).

$$
c r=\frac{R_{S T}}{R_{T}}
$$

- Where the subscript ST refers to the student (participant), and T to the teacher (criterion).
- In the criterion the conceptual nucleus was determined as the one formed by those concepts that are more related, and
- We calculated the $\boldsymbol{c}$ index of correspondence between the number of concepts in the conceptual core of the criterion and the number of core concepts that are present in the responses of the participants, that is,

$$
c=\frac{C_{S T c}}{C_{T c}}
$$

- The quality of knowledge defined and measured by the partial or complete presence of certain concepts connected in a certain way $\boldsymbol{q}$ together with the correspondence index $\boldsymbol{c}$ in the nucleus, respond to a question of quality


## How correct?

- It is an indicator of the degree of accuracy of the response and weighted by the density $\boldsymbol{d}$


## What good is said?

- It is a measure of the consistency that a global quality index $Q$ provides for the participant's response.

$$
q=c c \cdot c r \quad Q=\frac{c+q}{d}
$$

## SYNTHESIS

| Indices Framework | Density (d) | Conceptual correpondence (cc) | Correspondence in relationships (cr) | Correspondence in the core (c) | Quality index (q) | Global quality index (Q) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Notional | $d>2$ | cc<0.25 | cr $<0.25$ | $c<0.25$ | $q<0.0625$ | $0<=Q<=0.1562$ |
| Refrential | $1.38<\mathrm{d}<=2$ | $0.25<=$ cc < 0.5 | $0.25<=\mathrm{cr}<0.5$ | $0.25<=c<0.5$ | $0.0625<=q<0.25$ | $0.1562<=Q<0.543$ |
| Conceptual | d < $=1.38$ | $0.5<=$ cc | $0.5<=\mathrm{cr}$ | $0.5<=$ c | $q>=0.25$ | $Q<=0.543$ |

$$
\begin{array}{cc}
d=\frac{C}{R} \quad c c=\frac{C_{T S}}{C_{T}} \quad c r=\frac{R_{S T C}}{R_{T C}} \quad c=\frac{C_{S T c}}{C_{T c}} \\
q=c c \cdot c r \\
Q=\frac{q+c}{d}
\end{array}
$$

## RESULTS AND DISCUSSION



## SELECTED EXAMPLES







## COMPARISON CRITERION VS P2



MAP Criterion




27

## COMPARISON CRITERION VS P11



## COMPARISON CRITERION VS P13

MAP Criterion


## COMPARISON CRITERION VS P17



## PARTICIPANTS RESULTS

|  | C | R | d | cc | cr | c | q | Q | Conclusion |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| P11 | 23 | 16 | 1.4375 | 0.1500 | 0.2500 | 0.4000 | 0.0375 | 0.3043 | referential |
| P2 | 11 | 8 | 1.3750 | 0.1500 | 0.1667 | 0.4000 | 0.0250 | 0.3091 | referential |
| P13 | 15 | 13 | 1.1538 | 0.1500 | 0.0556 | 0.4000 | 0.0083 | 0.3539 | referential |
| P17 | 6 | 10 | 0.6000 | 0.4000 | 0.3333 | 0.6000 | 0.1333 | 1.2222 | referential $(+)$ |

## RESULTS OF 19 PARTICIPANTS

TABLE 2 Classification of the conceptual organization
Data organized increasingly according to the Q index

|  |  | Correspondencia conceptual | Conceptual correspondence | Correspondence in the core (c) | Quality (q) | Global quality ( Q ) | Conclusion |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Participant | Density (d) | (cc) | (cr) | (c) | (q) | (Q) | framework |
| 1 | 2.0000 | 0.1500 | 0.3333 | 0.3333 | 0.0500 | 0.1917 | referential (-) |
| 11 | 1.4375 | 0.1500 | 0.2500 | 0.4000 | 0.0375 | 0.3043 | referential (-) |
| 2 | 1.3750 | 0.1500 | 0.1667 | 0.4000 | 0.0250 | 0.3091 | referential (-) |
| 5 | 1.1429 | 0.1500 | 0.3333 | 0.3333 | 0.0500 | 0.3354 | referential (-) |
| 9 | 2.0000 | 0.3000 | 0.6000 | 0.5000 | 0.1800 | 0.3400 | referential |
| 7 | 1.1100 | 0.2000 | 0.2222 | 0.3333 | 0.0444 | 0.3403 | notional |
| 4 | 2.5000 | 0.3500 | 0.5385 | 0.6667 | 0.1885 | 0.3421 | referential |
| 13 | 1.1538 | 0.1500 | 0.0556 | 0.4000 | 0.0083 | 0.3539 | referential |
| 6 | 1.2000 | 0.1500 | 0.6667 | 0.3333 | 0.1000 | 0.3611 | referential |
| 3 | 1.8300 | 0.2500 | 0.3000 | 0.6667 | 0.0750 | 0.4053 | referential |
| 17 | 1.3000 | 0.4000 | 0.3333 | 0.6000 | 0.1333 | 0.5641 | referential (+) |
| 12 | 1.2857 | 0.3000 | 0.2143 | 0.6667 | 0.0643 | 0.5685 | referential (+) |
| 14 | 1.3300 | 0.3000 | 0.4615 | 0.6667 | 0.1385 | 0.6054 | referential (+) |
| 15 | 1.4000 | 0.5000 | 0.3750 | 0.6667 | 0.1875 | 0.6101 | referential (+) |
| 8 | 0.8462 | 0.2500 | 0.0769 | 0.5000 | 0.0192 | 0.6136 | referential |
| 19 | 1.0000 | 0.4000 | 0.3636 | 0.5000 | 0.1455 | 0.6455 | referential (+) |
| 10 | 1.0769 | 0.3000 | 0.2308 | 0.6667 | 0.0692 | 0.6833 | referential (+) |
| 16 | 1.1667 | 0.3500 | 0.6250 | 0.6667 | 0.2188 | 0.7589 | referential (+) |
| 18 | 1.0000 | 0.3000 | 0.4167 | 0.6667 | 0.1250 | 0.7917 | referential (+) |

Framework notional
Framework referential $\square$
$\qquad$
Framework Conceptual


## REGARDING THE RESEARCH QUESTION

- How does the learner acquire mathematical concepts and how do they organize them to produce acceptable responses in the school environment


## CONCLUSIONS

- The participants' conceptual organizations 2 and 11 are close to the notional framework, mainly due to the

1. the conceptual (0.15) and relational (0.17), for P2, and
2. $(0.15)$ and $(0.25)$, for P11 correspondence indexes as well as
3. the quality indices $(0.025)$ and $(0.0375)$ belong to that framework,

- which means that they have poor conceptual structures conceptual of a minimum part of the fundamentals.
- Participant 13 is located in the referential framework but very close to the notional too, because $\boldsymbol{c r}$ and the correspondence index in the nucleus $\boldsymbol{c}$ indicates that it has almost than half of the basics, a situation that allows it to be located there and not in the notional framework.
- The participant's conceptual organization 17 corresponds to the referential framework, mainly due to the fact that the conceptual ( 0.4 ) and relational ( 0.33 ) correspondence indexes as well as the quality index (0.133) belong to that framework, which means that it has a poor structure conceptual of a minimum part of the fundamentals.


## CONCLUSIONS

- Five participants $(1,5,6,7$, and 11$)$ have a poor structure but a reasonable portion of the fundamentals (core concepts) and an equally reasonable number of concepts and have produced them in a reasonably coherent discursive mode when responding.
- Eight participants $(10,12,14,15,16,17,18$ and 19) have a conceptual organization belonging to the referential framework, but very close to the conceptual framework, that is, they have the foundations, an acceptable number of concepts and acceptable connections between them and produced in a coherent discourse when they respond to the item.


## CONCLUSIONS

- The density and the correspondence index in the core denote that they write well the basics and this has an important weight in the global quality index.
- As can be seen, the overall quality of the discursive response is quite good, 9 of 19 participants have an overall quality index greater than 0.543.
- However, they are not located in the conceptual framework due in part to the poor conceptual structure and low quality in the correspondence of concepts and relationships.


## CONCLUSIVE NOTE

- The differences between the responses of the participants are attributed to the quality and depth of the discussion derived from the interaction.


## REFERENCES

- Balderas, P. (1996) Representación del concepto de cambio en ambientes computacionales. En M. Campos y R. Ruiz (eds.), Problemas de acceso al conocimiento y enseñanza de las ciencias, México, IIMAS-UNAM, ISBN: 968-36-4927-0, 137-158.
- Balderas, P. (1998) La representación y el razonamiento visual en la enseñanza de la matemática. Tesis doctoral, UNAM.
- Balderas, P. (2018) ) Digraphs for modeling systems' representation of mathematical knowledge 29th European Conference on Operational Research (EURO2018), Valencia, España.
- Bloome, D. (1992) Interacción e intertextualidad en el estudio de la lectoescritua en las aulas: el microanálisis como una tarea teórica. En M. Rueda y M. Campos (eds.), Investigación Etnográfica en Educación, México, UNAM, p. 123-180.
- Campos, M.A. y Gaspar, S. (1995) The Propositional Analysis Model: Semantic Analysis of Correspondence in Knowledge Construction. Reportes de investigación. México, IIMAS-UNAM, (5), 49, octubre.
- Harary, F., Norman, R. y Cartwright, D. (1965) Structural Models: An introduction to the Theory of Directed Graphs. New York, John Wiley \& Sons, Inc.
- Lincoln, Y. y Guba, E. (1985) Naturalistic inquiry. Newbury Park, Sage Publications.
- Light, G. and Cox, R. (2001) Learning \& Teaching in Higher Education. London, Paul Chapman Publishing.


## THANKS FOR YOUR ATTENTION

