# UNIVERSIDAD NACIONAL AUTONOMA DE MEXICO

EXAMEN: ______________________

PROFESOR: ______________________

MATERIA: ______________________

NOMBRE DEL ALUMNO: ______________________

TAREA DE E.D. Entrega 25 de marzo

RESOLVER LAS SIGS. EC. DIF., MEDIANTE EL MÉTODO DE COEFICIENTES INDET.

1) $y'' - 2y' + 5y = e^x \operatorname{sen} x \quad \text{—(A)}$

$y_h: \quad y'' - 2y' + 5y = 0$

$(D^2 - 2D + 5)y = 0$

$\lambda^2 - 2\lambda + 5 = 0$

$\lambda_{1,2} = 1 \pm 2i \; ; \; \lambda_1 = 1 + 2i, \; \lambda_2 = 1 - 2i$

$$y_h = C_1 e^x \cos 2x + C_2 e^x \operatorname{sen} 2x$$

$y_p: \quad q(x) = e^x \operatorname{sen} x$

$\alpha = 1, \; \beta = 1, \; n = 1$

$P(D) = D^2 - 2D + 2$

Aplicando $P(D)$ en (A)

$(D^2 - 2D + 2)(D^2 - 2D + 5)y = 0$

$\lambda_{1,2} = 1 \pm 2i$

$\lambda_{3,4} = 1 \pm i$

$$y = C_1 e^x \cos 2x + C_2 e^x \operatorname{sen} 2x + \underbrace{C_3 e^x \cos x + C_4 e^x \operatorname{sen} x}_{y_p}$$

$y_p = C_3 e^x \cos x + C_4 e^x \operatorname{sen} x$

$y'_p = -C_3 e^x \operatorname{sen} x + C_3 e^x \cos x + C_4 e^x \cos x + C_4 e^x \operatorname{sen} x$

$y''_p = -C_3 e^x \cos x + (-C_3 e^x \operatorname{sen} x) + (-C_3 e^x \operatorname{sen} x + C_3 e^x \cos x) + \longrightarrow *$

$* \longrightarrow \; -C_4 e^x \operatorname{sen} x + C_4 e^x \cos x + C_4 e^x \cos x + C_4 e^x \operatorname{sen} x$

$y''_p = -2C_3 e^x \operatorname{sen} x + 2C_4 e^x \cos x$

Sust. $y'_p$ y $y''_p$ en (A)

$-2C_3 e^x \operatorname{sen} x + 2C_4 e^x \cos x + 2C_3 e^x \operatorname{sen} x - 2C_3 e^x \cos x - 2C_4 e^x \cos x - 2C_4 e^x \operatorname{sen} x + 5C_3 e^x \cos x + 5C_4 e^x \operatorname{sen} x = e^x \operatorname{sen} x$

$3C_3 e^x \cos x + 3C_4 e^x \operatorname{sen} x = e^x \operatorname{sen} x$

$3C_3 = 0 \implies C_3 = 0$

$3C_4 = 1 \implies C_4 = \frac{1}{3}$

$y_p = \frac{1}{3} e^x \operatorname{sen} x$

$y_g = y_h + y_p$

$$y = C_1 e^x \cos 2x + C_2 e^x \operatorname{sen} 2x + \frac{1}{3} e^x \operatorname{sen} x$$

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CONTINÚA SOLUCIÓN DE TAREA (25 DE MARZO)

2) $y''' - 3y'' + 3y' - y = e^x - x + 16 \quad -(A)$

$y_h$: $y''' - 3y'' + 3y' - y = 0$

$(D^3 - 3D^2 + 3D - 1)y = 0$

$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$

$(\lambda - 1)^3 = 0$

Las raíces son: $\lambda_1 = \lambda_2 = \lambda^3 = 1$

$y_h = C_1 e^x + C_2 x e^x + C_3 x^2 e^x$

$y_p$: $q(x) = e^x - x + 16$

$P(D) = (D-1)(D^2)$

$(D-1)D^2(D-1)^3 y = 0$

$(\lambda-1)\lambda^2(\lambda-1)^3 = 0$; $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1$

$\lambda_5 = \lambda_6 = 0$

$y = C_1 e^x + C_2 x e^x + C_3 x^2 e^x + C_4 x^3 e^x + C_5 + C_6 x$

$y_p = C_4 x^3 e^x + C_5 + C_6 x$

Se calcula $y'_p$, $y''_p$, $y'''_p$.

$y'_p = C_4 x^3 e^x + 3C_4 x^2 e^x + C_6$

$y''_p = C_4 x^3 e^x + 6C_4 x^2 e^x + 6C_4 x e^x$

$y'''_p = C_4 x^3 e^x + 9C_4 x^2 e^x + 18 C_4 x e^x + 6C_4 e^x$

Sustituyendo en (A) y efectuando operaciones para simplificar:

RESULTA:

$6C_4 e^x + 3C_6 - C_5 - C_6 x = e^x - x + 16$

POR IGUALDAD DE POLINOMIOS:

$6C_4 = 1 \Rightarrow C_4 = \frac{1}{6}$

$-C_6 = -1 \Rightarrow C_6 = 1$

$3C_6 - C_5 = 16 \Rightarrow C_5 = -13$

$y_p = \frac{1}{6} x^3 e^x + x - 13$

FINALMENTE:

$y_G = y_h + y_p$

$y_G = C_1 e^x + C_2 x e^x + C_3 x^2 e^x + \frac{1}{6} x^3 e^x + x - 13$