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Deducción del método de Newton Raphson para 3 ecuaciones con 3 incógnitas

$$u_{i+1} = u_i + (x_{i+1} - x_i)\frac{\partial u_i}{\partial x} + (y_{i+1} - y_i)\frac{\partial u_i}{\partial y} + (z_{i+1} - z_i)\frac{\partial u_i}{\partial z}$$

$$v_{i+1} = v_i + (x_{i+1} - x_i)\frac{\partial v_i}{\partial x} + (y_{i+1} - y_i)\frac{\partial v_i}{\partial y} + (z_{i+1} - z_i)\frac{\partial v_i}{\partial z}$$

$$w_{i+1} = w_i + (x_{i+1} - x_i)\frac{\partial w_i}{\partial x} + (y_{i+1} - y_i)\frac{\partial w_i}{\partial y} + (z_{i+1} - z_i)\frac{\partial w_i}{\partial z}$$

Entonces tenemos el sistema donde: (x_{i+1}), (y_{i+1}), (z_{i+1}) son las incógnitas

$$\begin{cases} \frac{\partial u_i}{\partial x}(x_{i+1}) + \frac{\partial u_i}{\partial y}(y_{i+1}) + \frac{\partial u_i}{\partial z}(z_{i+1}) = -u_i + x_i\frac{\partial u_i}{\partial x} + y_i\frac{\partial u_i}{\partial y} + z_i\frac{\partial u_i}{\partial z} \\ \frac{\partial v_i}{\partial x}(x_{i+1}) + \frac{\partial v_i}{\partial y}(y_{i+1}) + \frac{\partial v_i}{\partial z}(z_{i+1}) = -v_i + x_i\frac{\partial v_i}{\partial x} + y_i\frac{\partial v_i}{\partial y} + z_i\frac{\partial v_i}{\partial z} \\ \frac{\partial w_i}{\partial x}(x_{i+1}) + \frac{\partial w_i}{\partial y}(y_{i+1}) + \frac{\partial w_i}{\partial z}(z_{i+1}) = -w_i + x_i\frac{\partial w_i}{\partial x} + y_i\frac{\partial w_i}{\partial y} + z_i\frac{\partial w_i}{\partial z} \end{cases}$$

Calculando el sistema de ecuaciones

$$x_{i+1} = \frac{\begin{vmatrix} -u_i + x_i\frac{\partial u_i}{\partial x} + y_i\frac{\partial u_i}{\partial y} + z_i\frac{\partial u_i}{\partial z} & \frac{\partial u_i}{\partial y} & \frac{\partial u_i}{\partial z} \\ -v_i + x_i\frac{\partial v_i}{\partial x} + y_i\frac{\partial v_i}{\partial y} + z_i\frac{\partial v_i}{\partial z} & \frac{\partial v_i}{\partial y} & \frac{\partial v_i}{\partial z} \\ -w_i + x_i\frac{\partial w_i}{\partial x} + y_i\frac{\partial w_i}{\partial y} + z_i\frac{\partial w_i}{\partial z} & \frac{\partial w_i}{\partial y} & \frac{\partial w_i}{\partial z} \end{vmatrix}}{\begin{vmatrix} \frac{\partial u_i}{\partial x} & \frac{\partial u_i}{\partial y} & \frac{\partial u_i}{\partial z} \\ \frac{\partial v_i}{\partial x} & \frac{\partial v_i}{\partial y} & \frac{\partial v_i}{\partial z} \\ \frac{\partial w_i}{\partial x} & \frac{\partial w_i}{\partial y} & \frac{\partial w_i}{\partial z} \end{vmatrix}}$$

Desarrollando el determinante superior (por cofactores en la primera columna)

$$- u_i \left(\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z}\right) + x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z}\right) + y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z}\right) + z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z}\right) + \cdots$$

$$u_i \left(\frac{\partial w_i}{\partial y}\frac{\partial v_i}{\partial z}\right) - x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial w_i}{\partial y}\frac{\partial v_i}{\partial z}\right) - y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial w_i}{\partial y}\frac{\partial v_i}{\partial z}\right) - z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial w_i}{\partial y}\frac{\partial v_i}{\partial z}\right)$$

$$\ldots + v_i \left(\frac{\partial u_i}{\partial y}\frac{\partial w_i}{\partial z}\right) - x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial u_i}{\partial y}\frac{\partial w_i}{\partial z}\right) - y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial u_i}{\partial y}\frac{\partial w_i}{\partial z}\right) - z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial u_i}{\partial y}\frac{\partial w_i}{\partial z}\right) + \ldots\ldots..$$

$$\ldots. + -v_i \left(\frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z}\right) + x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z}\right) + y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z}\right) + z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z}\right) + \ldots\ldots$$

$$\ldots..-w_i \left(\frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\right) + x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\right) + y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\right) + z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\right) + \ldots\ldots$$

$$+ \ldots\, w_i \left(\frac{\partial v_i}{\partial y}\frac{\partial u_i}{\partial z}\right) - x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial v_i}{\partial y}\frac{\partial u_i}{\partial z}\right) - y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial v_i}{\partial y}\frac{\partial u_i}{\partial z}\right) - z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial v_i}{\partial y}\frac{\partial u_i}{\partial z}\right)$$

Desarrollando el determinante inferior por la regla Sarrus (este determinante es necesario calcularlo una sola vez)

$$\left|\begin{array}{ccc|cc} \frac{\partial u_i}{\partial x} & \frac{\partial u_i}{\partial y} & \frac{\partial u_i}{\partial z} & \frac{\partial u_i}{\partial x} & \frac{\partial u_i}{\partial y} \\ \frac{\partial v_i}{\partial x} & \frac{\partial v_i}{\partial y} & \frac{\partial v_i}{\partial z} & \frac{\partial v_i}{\partial x} & \frac{\partial v_i}{\partial y} \\ \frac{\partial w_i}{\partial x} & \frac{\partial w_i}{\partial y} & \frac{\partial w_i}{\partial z} & \frac{\partial w_i}{\partial x} & \frac{\partial w_i}{\partial y} \end{array}\right.$$

Luego tenemos el resultado del determinante inferior

$$\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}$$

Tenemos que:

$$x_{i+1} = \frac{\left[u_i\left(\frac{\partial w_i}{\partial y}\frac{\partial v_i}{\partial z} - \frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z}\right) + v_i\left(\frac{\partial u_i}{\partial y}\frac{\partial w_i}{\partial z} - \frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z}\right) + w_i\left(\frac{\partial v_i}{\partial y}\frac{\partial u_i}{\partial z} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\right)\right] + x_i\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right)}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}}$$

$$x_{i+1} = \qquad 1$$

$$x_i\frac{\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right)}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}} + \ldots$$

$$\ldots + \frac{\left[u_i\left(\frac{\partial w_i}{\partial y}\frac{\partial v_i}{\partial z} - \frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z}\right) + v_i\left(\frac{\partial u_i}{\partial y}\frac{\partial w_i}{\partial z} - \frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z}\right) + w_i\left(\frac{\partial v_i}{\partial y}\frac{\partial u_i}{\partial z} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\right)\right]}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}}$$

Luego de factorizar el por x_i queda:

$$x_{i+1} = x_i - \frac{\left[u_i\left(\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} - \frac{\partial w_i}{\partial y}\frac{\partial v_i}{\partial z}\right) + v_i\left(\frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z} - \frac{\partial u_i}{\partial y}\frac{\partial w_i}{\partial z}\right) + w_i\left(\frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z} - \frac{\partial v_i}{\partial y}\frac{\partial u_i}{\partial z}\right)\right]}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}}$$

$$y_{i+1} = \frac{\begin{vmatrix} \frac{\partial u_i}{\partial x} & -u_i + x_i\frac{\partial u_i}{\partial x} + y_i\frac{\partial u_i}{\partial y} + z_i\frac{\partial u_i}{\partial z} & \frac{\partial u_i}{\partial z} \\ \frac{\partial v_i}{\partial x} & -v_i + x_i\frac{\partial v_i}{\partial x} + y_i\frac{\partial v_i}{\partial y} + z_i\frac{\partial v_i}{\partial z} & \frac{\partial v_i}{\partial z} \\ \frac{\partial w_i}{\partial x} & -w_i + x_i\frac{\partial w_i}{\partial x} + y_i\frac{\partial w_i}{\partial y} + z_i\frac{\partial w_i}{\partial z} & \frac{\partial w_i}{\partial z} \end{vmatrix}}{\begin{vmatrix} \frac{\partial u_i}{\partial x} & \frac{\partial u_i}{\partial y} & \frac{\partial u_i}{\partial z} \\ \frac{\partial v_i}{\partial x} & \frac{\partial v_i}{\partial y} & \frac{\partial v_i}{\partial z} \\ \frac{\partial w_i}{\partial x} & \frac{\partial w_i}{\partial y} & \frac{\partial w_i}{\partial z} \end{vmatrix}}$$

Desarollando el determinante superior (por cofactores en la segunda columna)

$$u_i\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + x_i\frac{\partial u_i}{\partial x}\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + y_i\frac{\partial u_i}{\partial y}\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + z_i\frac{\partial u_i}{\partial z}\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + \cdots$$

$$-u_i\left(\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial z}\right) + x_i\frac{\partial u_i}{\partial x}\left(\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial z}\right) + y_i\frac{\partial u_i}{\partial y}\left(\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial z}\right) + z_i\frac{\partial u_i}{\partial z}\left(\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial z}\right) + \ldots\ldots$$

$$\ldots - v_i\left(\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + x_i\frac{\partial u_i}{\partial x}\left(\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + y_i\frac{\partial u_i}{\partial y}\left(\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + z_i\frac{\partial u_i}{\partial z}\left(\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + \ldots\ldots\ldots$$

$$\ldots\; +v_i\left(\frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z}\right) - x_i\frac{\partial u_i}{\partial x}\left(\frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z}\right) - y_i\frac{\partial u_i}{\partial y}\left(\frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z}\right) - z_i\frac{\partial u_i}{\partial z}\left(\frac{\partial w_i}{\partial y}\frac{\partial u_i}{\partial z}\right) + \ldots\ldots$$

$$\ldots\ldots +w_i\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\right) - x_i\frac{\partial u_i}{\partial x}\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\right) - y_i\frac{\partial u_i}{\partial y}\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\right) - z_i\frac{\partial u_i}{\partial z}\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\right) + \ldots\ldots$$

$$+ \ldots - w_i\left(\frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial z}\right) + x_i\frac{\partial u_i}{\partial x}\left(\frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial z}\right) + y_i\frac{\partial u_i}{\partial y}\left(\frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial z}\right) - z_i\frac{\partial u_i}{\partial z}\left(\frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial z}\right)$$

Como el determinante inferior es el mismo para todas las incógnitas se tiene que:

$$\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}$$

Luego:

$$y_{i+1} = \frac{\left[u_i\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z} - \frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial z}\right) + v_i\left(\frac{\partial w_i}{\partial x}\frac{\partial u_i}{\partial z} - \frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + w_i\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z} - \frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial z}\right)\right] + y_i\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right)}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}}$$

y_{i+1}=

$$y_i\frac{\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right)}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}} \; (1)$$

$+\ldots$

$$\frac{\left[u_i\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z} - \frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial z}\right) + v_i\left(\frac{\partial w_i}{\partial x}\frac{\partial u_i}{\partial z} - \frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + w_i\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z} - \frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial z}\right)\right]}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}}$$

Factorizando por y_i queda:

$$y_{i+1} = y_i - \frac{\left[u_i\left(\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial z} - \frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right) + v_i\left(\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial z} - \frac{\partial w_i}{\partial x}\frac{\partial u_i}{\partial z}\right) + w_i\left(\frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial z} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\right)\right]}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}}$$

$$z_{i+1} = \frac{\begin{vmatrix} \frac{\partial u_i}{\partial x} & \frac{\partial u_i}{\partial y} & -u_i + x_i \frac{\partial u_i}{\partial x} + y_i \frac{\partial u_i}{\partial y} + z_i \frac{\partial u_i}{\partial z} \\ \frac{\partial v_i}{\partial x} & \frac{\partial v_i}{\partial y} & -v_i + x_i \frac{\partial v_i}{\partial x} + y_i \frac{\partial v_i}{\partial y} + z_i \frac{\partial v_i}{\partial z} \\ \frac{\partial w_i}{\partial x} & \frac{\partial w_i}{\partial y} & -w_i + x_i \frac{\partial w_i}{\partial x} + y_i \frac{\partial w_i}{\partial y} + z_i \frac{\partial w_i}{\partial z} \end{vmatrix}}{\begin{vmatrix} \frac{\partial u_i}{\partial x} & \frac{\partial u_i}{\partial y} & \frac{\partial u_i}{\partial z} \\ \frac{\partial v_i}{\partial x} & \frac{\partial v_i}{\partial y} & \frac{\partial v_i}{\partial z} \\ \frac{\partial w_i}{\partial x} & \frac{\partial w_i}{\partial y} & \frac{\partial w_i}{\partial z} \end{vmatrix}}$$

Desarollando el determinante superior (en la tercera columna)

$$-u_i \left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y}\right) + x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y}\right) + y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y}\right) + z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y}\right) + \cdots$$

$$u_i \left(\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial y}\right) - x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial y}\right) - y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial y}\right) - z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial y}\right)$$

$$\ldots + v_i \left(\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial y}\right) - x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial y}\right) - y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial y}\right) - z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial y}\right) + \ldots\ldots$$

$$\ldots + -v_i \left(\frac{\partial w_i}{\partial x}\frac{\partial u_i}{\partial y}\right) + x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial w_i}{\partial x}\frac{\partial u_i}{\partial y}\right) + y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial w_i}{\partial x}\frac{\partial u_i}{\partial y}\right) + z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial w_i}{\partial x}\frac{\partial u_i}{\partial y}\right) + \ldots\ldots$$

$$\ldots - w_i \left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\right) + x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\right) + y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\right) + z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\right) + \ldots\ldots$$

$$+ \ldots w_i \left(\frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial y}\right) - x_i \frac{\partial u_i}{\partial x}\left(\frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial y}\right) - y_i \frac{\partial u_i}{\partial y}\left(\frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial y}\right) - z_i \frac{\partial u_i}{\partial z}\left(\frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial y}\right)$$

$$z_{i+1} = \frac{\left[u_i\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial y}\right) + v_i\left(\frac{\partial w_i}{\partial x}\frac{\partial u_i}{\partial y} - \frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial y}\right) + w_i\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y} - \frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial y}\right)\right] + z_i\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right)}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}}$$

$z_{i+1} =$

$$+ z_i \frac{\left(\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}\right)}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z} + \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x} + \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x} - \frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y} - \frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}} + \ldots$$

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$$+\cdots \frac{\left[u_i\left(\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial y}-\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y}\right)+v_i\left(\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial y}-\frac{\partial w_i}{\partial x}\frac{\partial u_i}{\partial y}\right)+w_i\left(\frac{\partial \mathrm{v}_i}{\partial x}\frac{\partial \mathrm{u}}{\partial y}-\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\right)\right]}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z}+\frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x}+\frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y}-\frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x}-\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y}-\frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}}$$

$$z_{i+1} = z_i - \frac{\left[u_i\left(\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y}-\frac{\partial w_i}{\partial x}\frac{\partial v_i}{\partial y}\right)+v_i\left(\frac{\partial w_i}{\partial x}\frac{\partial u_i}{\partial y}-\frac{\partial u_i}{\partial x}\frac{\partial w_i}{\partial y}\right)+w_i\left(\frac{\partial \mathrm{u}_i}{\partial x}\frac{\partial v_i}{\partial y}-\frac{\partial v_i}{\partial x}\frac{\partial u_i}{\partial y}\right)\right]}{\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial z}+\frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial x}+\frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial y}-\frac{\partial u_i}{\partial z}\frac{\partial v_i}{\partial y}\frac{\partial w_i}{\partial x}-\frac{\partial u_i}{\partial x}\frac{\partial v_i}{\partial z}\frac{\partial w_i}{\partial y}-\frac{\partial u_i}{\partial y}\frac{\partial v_i}{\partial x}\frac{\partial w_i}{\partial z}}$$

Problema 6.12

Determine las raíces de las siguientes ecuaciones no lineales, simultáneas por medio del método de Newton- Raphson:

Y = $-x^2 + x + 0.75$

$y + 5xy = x^2$

$\frac{\partial u_i}{\partial x} = 2x - 1$ =1.4 $\qquad \frac{\partial u_i}{\partial y} = 1$

$\frac{\partial v_i}{\partial x} = 5y - 2x$ =3.6 $\qquad \frac{\partial v_i}{\partial y} = 1 + 5x$ = 7

$u_i = -(1.2) - (1.2)^2 + (1.2) + 0.75$ =0.69

$v_i = (1.2) + 5(1.2)(1.2) - (1.2)^2 = -6.96$

Iteración	x_i	y_i	x_{i+1}	y_{i+1}
0	1.2	1.2	1.54355	0.02903
1	1.54355	0.02903	1.39412	0.22287
2	1.39412	0.22287	1.37245	0.23929

ITERACION 0:

$\frac{\partial u}{\partial x} = 1.4 \quad \frac{\partial u}{\partial y} = 1$

$\frac{\partial v}{\partial x} = -3.6 \quad \frac{\partial v}{\partial y} = -7$

u_i= 0.69

v_i=-6.96

x_{i+1}=1.54355

y_{i+1}= 0.02903

ITERACION1:

$\frac{\partial u}{\partial x} = 2.0871 \quad \frac{\partial u}{\partial y} = 1$

$\frac{\partial v}{\partial x}: 2.94195 \quad \frac{\partial v}{\partial y} = -8.71775$

u_i =0.11803

v_i =2.12947

x_{i+1}= 1.39412

y_{i+1}= 0.22287

ITERACION2:

$\frac{\partial u}{\partial x} = 1.78824 \quad \frac{\partial u}{\partial y} = 1$

$\frac{\partial v}{\partial x} : 1.67389 \quad \frac{\partial v}{\partial y} = -7.9706$

u_i =0.02232

v_i= 0.16716

x_{i+1}= 1.37245 $\quad y_{i+1}$= 0.23929

Problema#6.13

6.13 Encuentre las raíces de las ecuaciones simultáneas que siguen:

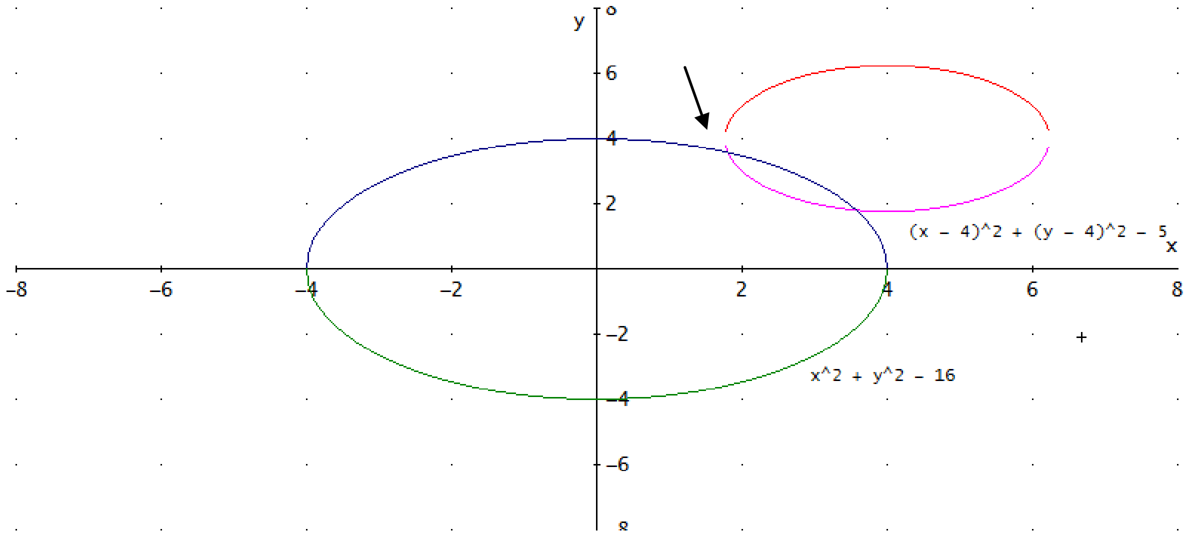
$(x-4)^2 + (y-4)^2 = 5$

$x^2 + y^2 = 16$

Entonces se Calculan las derivadas parciales.

$\frac{\partial u_i}{\partial x} = 2x - 8 \qquad \frac{\partial u_i}{\partial y} = 2y - 8$

$\frac{\partial v_i}{\partial x} = 2\text{x} \qquad \frac{\partial v_i}{\partial y} = 2\text{y}$



En la imagen se muestran 2 puntos de intersección se utiliza el método de Newton Raphson para encontrar la raíz más cercana a la interseccion.

Tomo x = 2 ; y=4

Interccion# 0

$\frac{\partial u_i}{\partial x} = 2(2) - 8 = -4$

$\frac{\partial u_i}{\partial y} = 2(4) - 8 = 0$

$\frac{\partial v_i}{\partial x} = 2(2) = 4$

$\frac{\partial v_i}{\partial y} = 2(4) = 8$

El determinante es: (-4) (8)-(4) (0)=-32

$u_i = [(x-4)^2 + (y-4)^2] - 5 \Rightarrow [(2-4)^2 + (4-4)^2] - 5 = -1$

$v_i = x^2 + y^2 - 16 \Rightarrow (2)^2 + (4)^2 - 16$= 4

Luego reemplazo en la ecuación los valores obtenidos:

$$x = 2 - \frac{-1(8) - (4)(0)}{-32} = 1.7500 \qquad y = 4 - \frac{4(-4) - (-1)(4)}{-32} = 3.6250$$

Interaccion#1

$\frac{\partial u_i}{\partial x} = 2(1.75) - 8 = -4.5$

$\frac{\partial u_i}{\partial y} = 2(3.625) - 8 = -0.75$

$\frac{\partial v_i}{\partial x} = 2(1.75) = 3.5$

$\frac{\partial v_i}{\partial y} = 2(3.625) = 7.25$

$u_i = [(x-4)^2 + (y-4)^2] - 5 \Rightarrow [(1.75-4)^2 + (3.625-4)^2] - 5 = 0.2031$

$v_i = x^2 + y^2 - 16 \Rightarrow (1.75)^2 + (3.625)^2 - 16$= 0.2031

Luego reemplazo en la ecuación los valores obtenidos:

$$x = 1.75 - \frac{(0.2031)(7.25) - (0.2031)(-0.75)}{(-4.5)(7.25) - (3.5)(-0.75)} = 1.8041$$

$$y = 4 - \frac{(0.2031)(-4.5) - (0.2031)(3.5)}{(-4.5)(7.25) - (3.5)(-0.75)} = 3.5708$$

Interación#2

$\frac{\partial u_i}{\partial x} = 2(1.8041) - 8 = -4.3918$

$\frac{\partial u_i}{\partial y} = 2(3.5708) - 8 = -0.8584$

$\frac{\partial v_i}{\partial x} = 2(1.8041) = 3.55708$

$\frac{\partial v_i}{\partial y} = 2(3.5708) = 7.1416$

$u_i = [(x-4)^2 + (y-4)^2] - 5 \Rightarrow [(1.8041-4)^2 + (3.5708-4)^2] - 5 = \ 0.0062$

$v_i = x^2 + y^2 - 16 \Rightarrow (1.8041)^2 + (3.5708)^2 - 16$= 0.0054

Luego reemplazo en la ecuación los valores obtenidos:

$$x = 1.8041 - \frac{(0.0062)(7.1416) - (0.0054)(-0.8584)}{(-4.3918)(7.1416) - (3.6082)(-0.8584)} = 1.8041$$

$$y = 4 - \frac{(0.0054)(-4.3918) - (0.0062)(3.6080)}{(-4.3918)(7.1416) - (3.6082)(-0.8584)} = 3.5708$$

Iteración	x_i	y_i	x_{i+1}	y_{i+1}
0	2	4	1.7500	3.6250
1	1.7500	3.6250	1.8041	3.5708
2	1.8041	3.5708	1.8041	3.5708

Problema#6.14

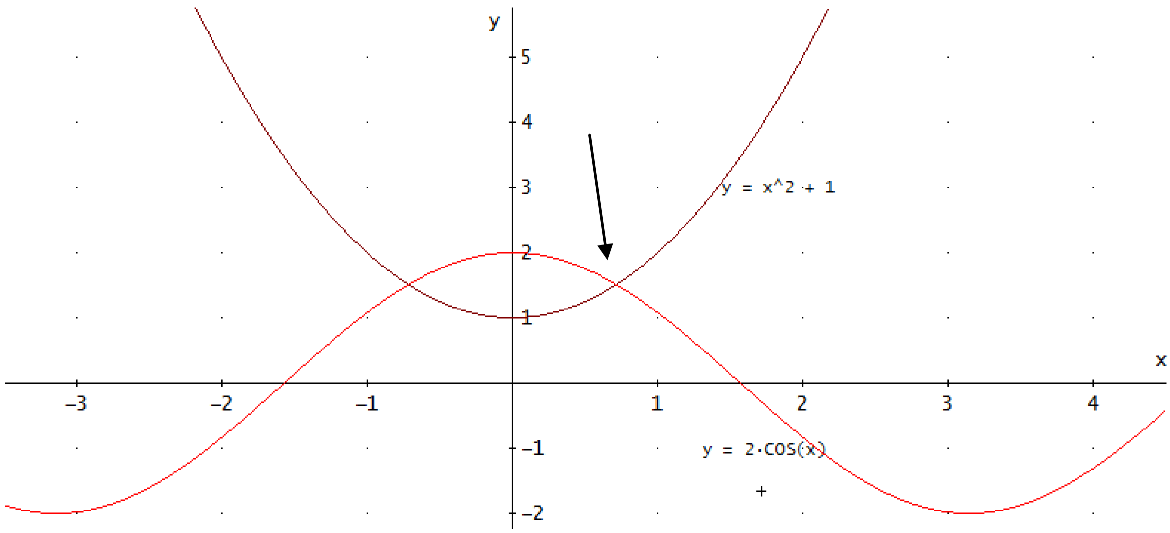
Repita el problema 6.13, excepto que

$y = x^2 + 1 \qquad y = 2\cos(x)$

Entonces se Calculan las derivadas parciales.

$\frac{\partial u_i}{\partial x} = 2x \qquad \frac{\partial u_i}{\partial y} = 1$

$\frac{\partial v_i}{\partial x} = -2\sin(\text{x}) \qquad \frac{\partial v_i}{\partial y} = 1$



Como en la imagen se muestran 2 puntos de intersección se utiliza el método de Newton Raphson para encontrar la raíz más cercana a la interseccion.

Tomo x=0.8; y=1.4

$$\frac{\partial u_i}{\partial x} = 2(0.8) = 1.6 \qquad \frac{\partial u_i}{\partial y} = 1$$

$$\frac{\partial v_i}{\partial x} = -2\sin(0.8) = -0.0279 \qquad \frac{\partial v_i}{\partial y} = 1$$

El determinante es: (1.6) (1)-(-0.0279) (1)=1.6279

u_i=$(0.8)^2 + 1 - (1.4) = 0.24$

$v_i = 2\cos(0.8) - (1.4)$ =0.5998

Luego reemplazo en la ecuación los valores obtenidos:

$$x = 0.8 - \frac{(0.24)(1) - (0.5958)((1)}{1.6279} \qquad y = 1.4 - \frac{(0.5988)(1.6) - (0.24)(-0.0279)}{1.6279}$$

$x = 1.0185 \qquad y = 0.80073$

$\frac{\partial u_i}{\partial x} = 2(1.0185) = 2.037$ $\frac{\partial u_i}{\partial y} = 1$

$\frac{\partial v_i}{\partial x} = -2\sin(1.0185) = -0.0356$ $\frac{\partial v_i}{\partial y} = 1$

El determinante es: (2.0370) (1)-(-0.0356) (1)=2.0726

u_i=$(1.0185)^2 + 1 - (0.8007) = 1.2366$

$v_i = 2\cos(1.0185) - (0.8007)$ =1.1990

reemplazo en la ecuación los valores obtenidos:

$$x = 1.0185 - \frac{(1.2366)(1) - (1.1990)(1)}{2.0726} = 1.0036$$

$$y = 0.8007 - \frac{(1.1990)(2.037) - (1.2366)(-0.0356)}{2.0726}$$ =-0.3989