ÁLGEBRA

OPERACIONES ARITMÉTICAS

$$a(b + c) = ab + ac \qquad \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a + c}{b} = \frac{a}{b} + \frac{c}{b} \qquad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

EXPONENTES Y RADICALES

$$x^m x^n = x^{m+n} \qquad \frac{x^m}{x^n} = x^{m-n}$$

$$(x^m)^n = x^{mn} \qquad x^{-n} = \frac{1}{x^n}$$

$$(xy)^n = x^n y^n \qquad \left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$x^{1/n} = \sqrt[n]{x} \qquad x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

$$\sqrt[n]{xy} = \sqrt[n]{x}\sqrt[n]{y} \qquad \sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}}$$

FACTORIZACIÓN ESPECIAL DE POLINOMIOS

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

TEOREMA DEL BINOMIO

$$(x + y)^2 = x^2 + 2xy + y^2 \qquad (x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2}x^{n-2}y^2$$

$$+ \cdots + \binom{n}{k}x^{n-k}y^k + \cdots + nxy^{n-1} + y^n$$

donde $\binom{n}{k} = \dfrac{n(n-1)\cdots(n-k+1)}{1 \cdot 2 \cdot 3 \cdot \cdots \cdot k}$

FÓRMULA CUADRÁTICA

Si $ax^2 + bx + c = 0$, entonces $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

DESIGUALDADES Y VALOR ABSOLUTO

Si $a < b$ y $b < c$, entonces $a < c$.

Si $a < b$, entonces $a + c < b + c$.

Si $a < b$ y $c > 0$, entonces $ca < cb$.

Si $a < b$ y $c < 0$, entonces $ca > cb$.

Si $a > 0$, entonces

$|x| = a$ significa $x = a$ o $x = -a$

$|x| < a$ significa $-a < x < a$

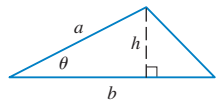
$|x| > a$ significa $x > a$ o $x < -a$

GEOMETRÍA

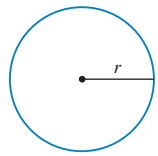
FÓRMULAS GEOMÉTRICAS

Fórmulas para área A, circunferencia C y volumen V:

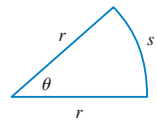
Triángulo
$A = \frac{1}{2}bh$
$= \frac{1}{2}ab \operatorname{sen}\theta$



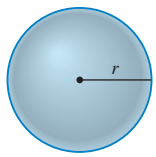
Círculo
$A = \pi r^2$
$C = 2\pi r$



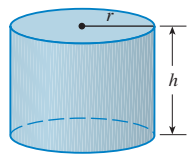
Sector circular
$A = \frac{1}{2}r^2\theta$
$s = r\theta$ (θ en radianes)



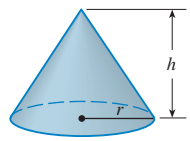
Esfera
$V = \frac{4}{3}\pi r^3$
$A = 4\pi r^2$



Cilindro
$V = \pi r^2 h$



Cono
$V = \frac{1}{3}\pi r^2 h$
$A = \pi r\sqrt{r^2 + h^2}$



FÓRMULAS DE DISTANCIA Y PUNTO MEDIO

Distancia entre $P_1(x_1, y_1)$ y $P_2(x_2, y_2)$:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Punto medio de $\overline{P_1P_2}$: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

LÍNEAS

Pendiente de la línea a través de $P_1(x_1, y_1)$ y $P_2(x_2, y_2)$:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Ecuación punto-pendiente de la línea a través de $P_1(x_1, y_1)$ con pendiente m:

$$y - y_1 = m(x - x_1)$$

Ecuación de la pendiente y la intersección de la línea con pendiente m e intersección de y en b

$$y = mx + b$$

CÍRCULOS

Ecuación de círculo con centro (h, k) y radio r:

$$(x - h)^2 + (y - k)^2 = r^2$$

TRIGONOMETRÍA

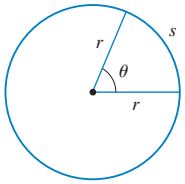
MEDICIÓN DE ÁNGULOS

π radianes $= 180°$

$1° = \dfrac{\pi}{180}$ rad $\qquad 1 \text{ rad} = \dfrac{180°}{\pi}$

$s = r\theta$

(θ en radianes)

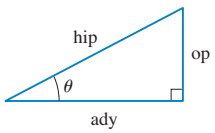


TRIGONOMETRÍA ÁNGULO RECTO

$\operatorname{sen}\theta = \dfrac{\text{op}}{\text{hip}} \qquad \csc\theta = \dfrac{\text{hip}}{\text{op}}$

$\cos\theta = \dfrac{\text{ady}}{\text{hip}} \qquad \sec\theta = \dfrac{\text{hip}}{\text{ady}}$

$\tan\theta = \dfrac{\text{op}}{\text{ady}} \qquad \cot\theta = \dfrac{\text{ady}}{\text{op}}$

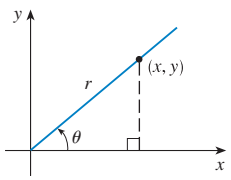


FUNCIONES TRIGONOMÉTRICAS

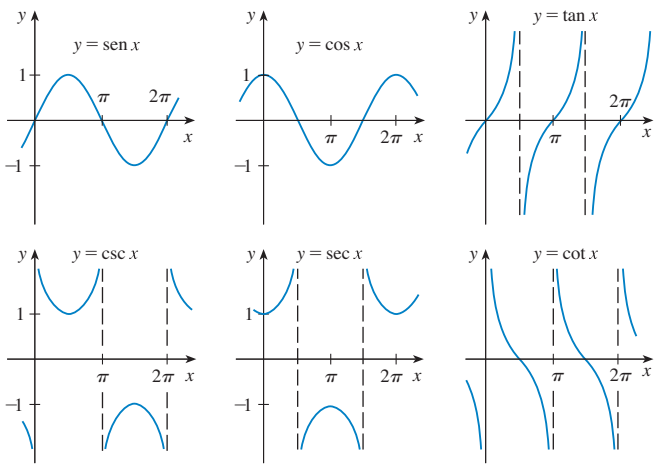
$\operatorname{sen}\theta = \dfrac{y}{r} \qquad \csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r} \qquad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x} \qquad \cot\theta = \dfrac{x}{y}$



GRÁFICAS DE FUNCIONES TRIGONOMÉTRICAS



FUNCIONES TRIGONOMÉTRICAS DE ÁNGULOS IMPORTANTES

θ	radianes	$\operatorname{sen}\theta$	$\cos\theta$	$\tan\theta$
0°	0	0	1	0
30°	$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
45°	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60°	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
90°	$\pi/2$	1	0	—

IDENTIDADES FUNDAMENTALES

$\csc\theta = \dfrac{1}{\operatorname{sen}\theta} \qquad \sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{\operatorname{sen}\theta}{\cos\theta} \qquad \cot\theta = \dfrac{\cos\theta}{\operatorname{sen}\theta}$

$\cot\theta = \dfrac{1}{\tan\theta} \qquad \operatorname{sen}^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$

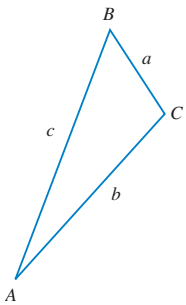
$\operatorname{sen}(-\theta) = -\operatorname{sen}\theta \qquad \cos(-\theta) = \cos\theta$

$\tan(-\theta) = -\tan\theta \qquad \operatorname{sen}\left(\dfrac{\pi}{2} - \theta\right) = \cos\theta$

$\cos\left(\dfrac{\pi}{2} - \theta\right) = \operatorname{sen}\theta \qquad \tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta$

LEY DE LOS SENOS

$$\frac{\operatorname{sen} A}{a} = \frac{\operatorname{sen} B}{b} = \frac{\operatorname{sen} C}{c}$$



LEY DE LOS COSENOS

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

FÓRMULAS DE SUMA Y RESTA

$\operatorname{sen}(x + y) = \operatorname{sen} x\cos y + \cos x\operatorname{sen} y$

$\operatorname{sen}(x - y) = \operatorname{sen} x\cos y - \cos x\operatorname{sen} y$

$\cos(x + y) = \cos x\cos y - \operatorname{sen} x\operatorname{sen} y$

$\cos(x - y) = \cos x\cos y + \operatorname{sen} x\operatorname{sen} y$

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x\tan y}$$

$$\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x\tan y}$$

FÓRMULAS DE ÁNGULOS DOBLES

$\operatorname{sen} 2x = 2\operatorname{sen} x\cos x$

$\cos 2x = \cos^2 x - \operatorname{sen}^2 x = 2\cos^2 x - 1 = 1 - 2\operatorname{sen}^2 x$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

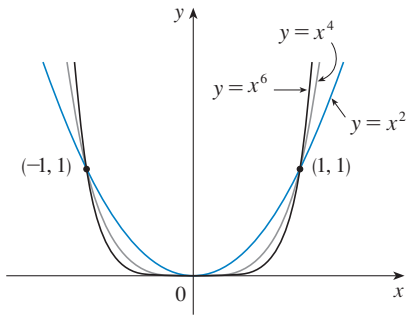
FÓRMULAS DE MEDIOS ÁNGULOS

$$\operatorname{sen}^2 x = \frac{1 - \cos 2x}{2} \qquad \cos^2 x = \frac{1 + \cos 2x}{2}$$

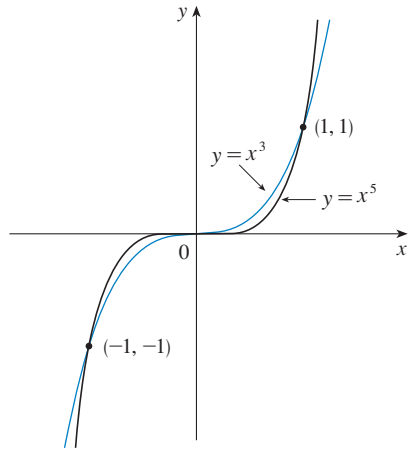
FUNCIONES ESPECIALES

FUNCIONES DE POTENCIAS $f(x) = x^a$

(i) $f(x) = x^n$, n es entero positivo

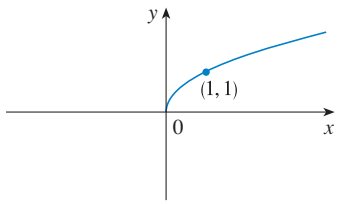


n par

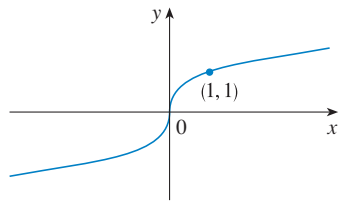


n impar

(ii) $f(x) = x^{1/n} = \sqrt[n]{x}$, n es entero positivo

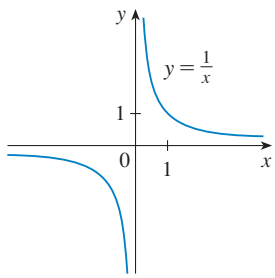


$f(x) = \sqrt{x}$



$f(x) = \sqrt[3]{x}$

(iii) $f(x) = x^{-1} = \dfrac{1}{x}$

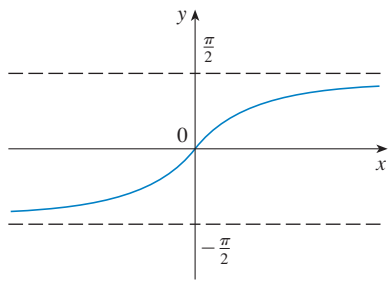


FUNCIONES TRIGONOMÉTRICAS INVERSAS

$\operatorname{arcsen} x = \operatorname{sen}^{-1} x = y \iff \operatorname{sen} y = x \quad \text{y} \quad -\dfrac{\pi}{2} \leqslant y \leqslant \dfrac{\pi}{2}$

$\arccos x = \cos^{-1} x = y \iff \cos y = x \quad \text{y} \quad 0 \leqslant y \leqslant \pi$

$\arctan x = \tan^{-1} x = y \iff \tan y = x \quad \text{y} \quad -\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$



$y = \tan^{-1} x = \arctan x$

$$\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$$

$$\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$$

FUNCIONES ESPECIALES

FUNCIONES EXPONENCIALES Y LOGARÍTMICAS

$\log_a x = y \iff a^y = x$

$\ln x = \log_e x, \quad \text{donde} \quad \ln e = 1$

$\ln x = y \iff e^y = x$

Ecuaciones de cancelación

$\log_a(a^x) = x \qquad a^{\log_a x} = x$

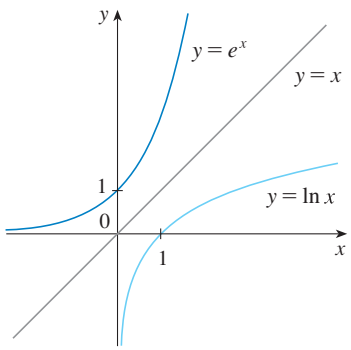
$\ln(e^x) = x \qquad e^{\ln x} = x$

Leyes de los logarítmos

1. $\log_a(xy) = \log_a x + \log_a y$

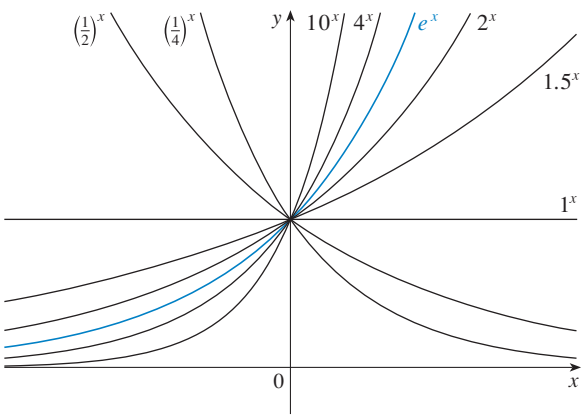
2. $\log_a\left(\dfrac{x}{y}\right) = \log_a x - \log_a y$

3. $\log_a(x^r) = r \log_a x$

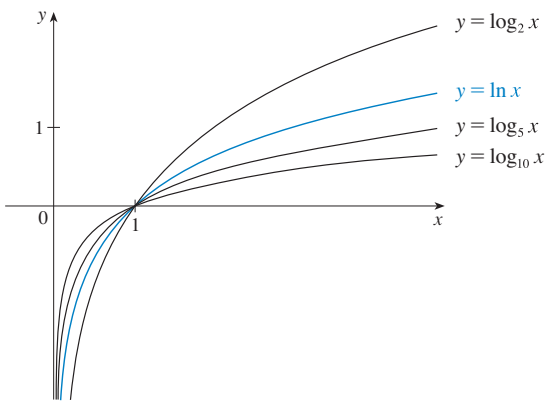


$\lim_{x \to -\infty} e^x = 0 \qquad \lim_{x \to \infty} e^x = \infty$

$\lim_{x \to 0^+} \ln x = -\infty \qquad \lim_{x \to \infty} \ln x = \infty$



Funciones exponenciales



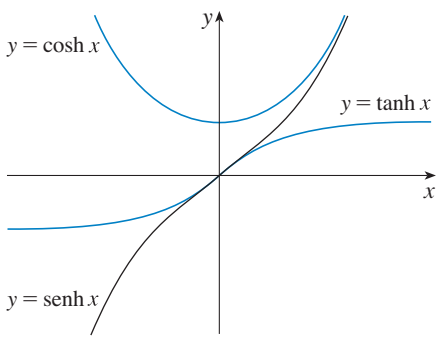
Funciones logarítmicas

FUNCIONES HIPERBÓLICAS

$\operatorname{senh} x = \dfrac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \dfrac{1}{\operatorname{senh} x}$

$\cosh x = \dfrac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \dfrac{1}{\cosh x}$

$\tanh x = \dfrac{\operatorname{senh} x}{\cosh x} \qquad \coth x = \dfrac{\cosh x}{\operatorname{senh} x}$



FUNCIONES HIPERBÓLICAS INVERSAS

$y = \operatorname{senh}^{-1} x \iff \operatorname{senh} y = x \qquad \operatorname{senh}^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$

$y = \cosh^{-1} x \iff \cosh y = x \quad \text{y} \quad y \geqslant 0 \qquad \cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$

$y = \tanh^{-1} x \iff \tanh y = x \qquad \tanh^{-1} x = \frac{1}{2} \ln\left(\dfrac{1 + x}{1 - x}\right)$

REGLAS DE DIFERENCIACIÓN

FÓRMULAS GENERALES

1. $\dfrac{d}{dx}(c) = 0$

2. $\dfrac{d}{dx}[cf(x)] = cf'(x)$

3. $\dfrac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

4. $\dfrac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

5. $\dfrac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Regla del producto)

6. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (Regla del cociente)

7. $\dfrac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Regla de la cadena)

8. $\dfrac{d}{dx}(x^n) = nx^{n-1}$ (Regla de potencias)

FUNCIONES EXPONENCIALES Y LOGARÍTMICAS

9. $\dfrac{d}{dx}(e^x) = e^x$

10. $\dfrac{d}{dx}(a^x) = a^x \ln a$

11. $\dfrac{d}{dx}\ln|x| = \dfrac{1}{x}$

12. $\dfrac{d}{dx}(\log_a x) = \dfrac{1}{x \ln a}$

FUNCIONES TRIGONOMÉTRICAS

13. $\dfrac{d}{dx}(\operatorname{sen} x) = \cos x$

14. $\dfrac{d}{dx}(\cos x) = -\operatorname{sen} x$

15. $\dfrac{d}{dx}(\tan x) = \sec^2 x$

16. $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$

17. $\dfrac{d}{dx}(\sec x) = \sec x \tan x$

18. $\dfrac{d}{dx}(\cot x) = -\csc^2 x$

FUNCIONES TRIGONOMÉTRICAS INVERSAS

19. $\dfrac{d}{dx}(\operatorname{sen}^{-1} x) = \dfrac{1}{\sqrt{1 - x^2}}$

20. $\dfrac{d}{dx}(\cos^{-1} x) = -\dfrac{1}{\sqrt{1 - x^2}}$

21. $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1 + x^2}$

22. $\dfrac{d}{dx}(\csc^{-1} x) = -\dfrac{1}{x\sqrt{x^2 - 1}}$

23. $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{x\sqrt{x^2 - 1}}$

24. $\dfrac{d}{dx}(\cot^{-1} x) = -\dfrac{1}{1 + x^2}$

FUNCIONES HIPERBÓLICAS

25. $\dfrac{d}{dx}(\operatorname{senh} x) = \cosh x$

26. $\dfrac{d}{dx}(\cosh x) = \operatorname{senh} x$

27. $\dfrac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

28. $\dfrac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

29. $\dfrac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

30. $\dfrac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

FUNCIONES HIPERBÓLICAS INVERSAS

31. $\dfrac{d}{dx}(\operatorname{senh}^{-1} x) = \dfrac{1}{\sqrt{1 + x^2}}$

32. $\dfrac{d}{dx}(\cosh^{-1} x) = \dfrac{1}{\sqrt{x^2 - 1}}$

33. $\dfrac{d}{dx}(\tanh^{-1} x) = \dfrac{1}{1 - x^2}$

34. $\dfrac{d}{dx}(\operatorname{csch}^{-1} x) = -\dfrac{1}{|x|\sqrt{x^2 + 1}}$

35. $\dfrac{d}{dx}(\operatorname{sech}^{-1} x) = -\dfrac{1}{x\sqrt{1 - x^2}}$

36. $\dfrac{d}{dx}(\coth^{-1} x) = \dfrac{1}{1 - x^2}$

TABLA DE INTEGRALES

FORMAS BÁSICAS

1. $\int u\,dv = uv - \int v\,du$

2. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C, \quad n \neq -1$

3. $\int \dfrac{du}{u} = \ln|u| + C$

4. $\int e^u\,du = e^u + C$

5. $\int a^u\,du = \dfrac{a^u}{\ln a} + C$

6. $\int \operatorname{sen} u\,du = -\cos u + C$

7. $\int \cos u\,du = \operatorname{sen} u + C$

8. $\int \sec^2 u\,du = \tan u + C$

9. $\int \csc^2 u\,du = -\cot u + C$

10. $\int \sec u \tan u\,du = \sec u + C$

11. $\int \csc u \cot u\,du = -\csc u + C$

12. $\int \tan u\,du = \ln|\sec u| + C$

13. $\int \cot u\,du = \ln|\operatorname{sen} u| + C$

14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

15. $\int \csc u\,du = \ln|\csc u - \cot u| + C$

16. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \operatorname{sen}^{-1}\dfrac{u}{a} + C$

17. $\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a}\tan^{-1}\dfrac{u}{a} + C$

18. $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\dfrac{u}{a} + C$

19. $\int \dfrac{du}{a^2 - u^2} = \dfrac{1}{2a}\ln\left|\dfrac{u+a}{u-a}\right| + C$

20. $\int \dfrac{du}{u^2 - a^2} = \dfrac{1}{2a}\ln\left|\dfrac{u-a}{u+a}\right| + C$

FORMAS QUE INVOLUCRAN $\sqrt{a^2 + u^2}$, $a > 0$

21. $\int \sqrt{a^2+u^2}\,du = \dfrac{u}{2}\sqrt{a^2+u^2} + \dfrac{a^2}{2}\ln\left(u + \sqrt{a^2+u^2}\right) + C$

22. $\int u^2\sqrt{a^2+u^2}\,du = \dfrac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \dfrac{a^4}{8}\ln\left(u + \sqrt{a^2+u^2}\right) + C$

23. $\int \dfrac{\sqrt{a^2+u^2}}{u}\,du = \sqrt{a^2+u^2} - a\ln\left|\dfrac{a + \sqrt{a^2+u^2}}{u}\right| + C$

24. $\int \dfrac{\sqrt{a^2+u^2}}{u^2}\,du = -\dfrac{\sqrt{a^2+u^2}}{u} + \ln\left(u + \sqrt{a^2+u^2}\right) + C$

25. $\int \dfrac{du}{\sqrt{a^2+u^2}} = \ln\left(u + \sqrt{a^2+u^2}\right) + C$

26. $\int \dfrac{u^2\,du}{\sqrt{a^2+u^2}} = \dfrac{u}{2}\sqrt{a^2+u^2} - \dfrac{a^2}{2}\ln\left(u + \sqrt{a^2+u^2}\right) + C$

27. $\int \dfrac{du}{u\sqrt{a^2+u^2}} = -\dfrac{1}{a}\ln\left|\dfrac{\sqrt{a^2+u^2}+a}{u}\right| + C$

28. $\int \dfrac{du}{u^2\sqrt{a^2+u^2}} = -\dfrac{\sqrt{a^2+u^2}}{a^2u} + C$

29. $\int \dfrac{du}{(a^2+u^2)^{3/2}} = \dfrac{u}{a^2\sqrt{a^2+u^2}} + C$

TABLA DE INTEGRALES

FORMAS QUE INVOLUCRAN $\sqrt{a^2 - u^2}$, $a > 0$

30. $\displaystyle\int \sqrt{a^2 - u^2}\, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$

31. $\displaystyle\int u^2\sqrt{a^2 - u^2}\, du = \frac{u}{8}(2u^2 - a^2)\sqrt{a^2 - u^2} + \frac{a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$

32. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u}\, du = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

33. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u^2}\, du = -\frac{1}{u}\sqrt{a^2 - u^2} - \operatorname{sen}^{-1}\frac{u}{a} + C$

34. $\displaystyle\int \frac{u^2\, du}{\sqrt{a^2 - u^2}} = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$

35. $\displaystyle\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

36. $\displaystyle\int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{1}{a^2 u}\sqrt{a^2 - u^2} + C$

37. $\displaystyle\int (a^2 - u^2)^{3/2}\, du = -\frac{u}{8}(2u^2 - 5a^2)\sqrt{a^2 - u^2} + \frac{3a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$

38. $\displaystyle\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2 - u^2}} + C$

FORMAS QUE INVOLUCRAN $\sqrt{u^2 - a^2}$, $a > 0$

39. $\displaystyle\int \sqrt{u^2 - a^2}\, du = \frac{u}{2}\sqrt{u^2 - a^2} - \frac{a^2}{2}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

40. $\displaystyle\int u^2\sqrt{u^2 - a^2}\, du = \frac{u}{8}(2u^2 - a^2)\sqrt{u^2 - a^2} - \frac{a^4}{8}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

41. $\displaystyle\int \frac{\sqrt{u^2 - a^2}}{u}\, du = \sqrt{u^2 - a^2} - a\cos^{-1}\frac{a}{|u|} + C$

42. $\displaystyle\int \frac{\sqrt{u^2 - a^2}}{u^2}\, du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln\left|u + \sqrt{u^2 - a^2}\right| + C$

43. $\displaystyle\int \frac{du}{\sqrt{u^2 - a^2}} = \ln\left|u + \sqrt{u^2 - a^2}\right| + C$

44. $\displaystyle\int \frac{u^2\, du}{\sqrt{u^2 - a^2}} = \frac{u}{2}\sqrt{u^2 - a^2} + \frac{a^2}{2}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

45. $\displaystyle\int \frac{du}{u^2\sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$

46. $\displaystyle\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2 - a^2}} + C$

TABLA DE INTEGRALES

FORMAS QUE INVOLUCRAN $a + bu$

47. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{1}{b^2}\left(a + bu - a\ln|a+bu|\right) + C$

48. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{1}{2b^3}\left[(a+bu)^2 - 4a(a+bu) + 2a^2\ln|a+bu|\right] + C$

49. $\displaystyle\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

50. $\displaystyle\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

51. $\displaystyle\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C$

52. $\displaystyle\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

53. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a+bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$

54. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$

55. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu} + C$

56. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2 + 3b^2u^2 - 4abu)\sqrt{a+bu} + C$

57. $\displaystyle\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C, \quad \text{si } a>0$

$$= \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C, \qquad \text{si } a<0$$

58. $\displaystyle\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$

59. $\displaystyle\int \frac{\sqrt{a+bu}}{u^2}\,du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a+bu}}$

60. $\displaystyle\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$

61. $\displaystyle\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int \frac{u^{n-1}\,du}{\sqrt{a+bu}}$

62. $\displaystyle\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$

TABLA DE INTEGRALES

FORMAS TRIGONOMÉTRICAS

63. $\int \operatorname{sen}^2 u\,du = \frac{1}{2}u - \frac{1}{4}\operatorname{sen} 2u + C$

64. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\operatorname{sen} 2u + C$

65. $\int \tan^2 u\,du = \tan u - u + C$

66. $\int \cot^2 u\,du = -\cot u - u + C$

67. $\int \operatorname{sen}^3 u\,du = -\frac{1}{3}(2 + \operatorname{sen}^2 u)\cos u + C$

68. $\int \cos^3 u\,du = \frac{1}{3}(2 + \cos^2 u)\operatorname{sen} u + C$

69. $\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$

70. $\int \cot^3 u\,du = -\frac{1}{2}\cot^2 u - \ln|\operatorname{sen} u| + C$

71. $\int \sec^3 u\,du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$

72. $\int \csc^3 u\,du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$

73. $\int \operatorname{sen}^n u\,du = -\frac{1}{n}\operatorname{sen}^{n-1} u \cos u + \frac{n-1}{n}\int \operatorname{sen}^{n-2} u\,du$

74. $\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u \operatorname{sen} u + \frac{n-1}{n}\int \cos^{n-2} u\,du$

75. $\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\,du$

76. $\int \cot^n u\,du = \frac{-1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\,du$

77. $\int \sec^n u\,du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$

78. $\int \csc^n u\,du = \frac{-1}{n-1}\cot u \csc^{n-2} u + \frac{n-2}{n-1}\int \csc^{n-2} u\,du$

79. $\int \operatorname{sen} au \operatorname{sen} bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} - \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$

80. $\int \cos au \cos bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} + \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$

81. $\int \operatorname{sen} au \cos bu\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

82. $\int u \operatorname{sen} u\,du = \operatorname{sen} u - u\cos u + C$

83. $\int u \cos u\,du = \cos u + u \operatorname{sen} u + C$

84. $\int u^n \operatorname{sen} u\,du = -u^n \cos u + n\int u^{n-1}\cos u\,du$

85. $\int u^n \cos u\,du = u^n \operatorname{sen} u - n\int u^{n-1}\operatorname{sen} u\,du$

86. $\int \operatorname{sen}^n u \cos^m u\,du = -\frac{\operatorname{sen}^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m}\int \operatorname{sen}^{n-2} u \cos^m u\,du$

$= \frac{\operatorname{sen}^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m}\int \operatorname{sen}^n u \cos^{m-2} u\,du$

FORMAS TRIGONOMÉTRICAS INVERSAS

87. $\int \operatorname{sen}^{-1} u\,du = u \operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$

88. $\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$

89. $\int \tan^{-1} u\,du = u\tan^{-1} u - \frac{1}{2}\ln(1+u^2) + C$

90. $\int u \operatorname{sen}^{-1} u\,du = \frac{2u^2-1}{4}\operatorname{sen}^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$

91. $\int u\cos^{-1} u\,du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$

92. $\int u\tan^{-1} u\,du = \frac{u^2+1}{2}\tan^{-1} u - \frac{u}{2} + C$

93. $\int u^n \operatorname{sen}^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\operatorname{sen}^{-1} u - \int \frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

94. $\int u^n \cos^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1} u + \int \frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

95. $\int u^n \tan^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1} u - \int \frac{u^{n+1}\,du}{1+u^2}\right], \quad n \neq -1$

TABLA DE INTEGRALES

FORMAS EXPONENCIALES Y LOGARÍTMICAS

96. $\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au} + C$

97. $\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$

98. $\int e^{au}\operatorname{sen} bu\,du = \frac{e^{au}}{a^2+b^2}(a\operatorname{sen} bu - b\cos bu) + C$

99. $\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\operatorname{sen} bu) + C$

100. $\int \ln u\,du = u\ln u - u + C$

101. $\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

102. $\int \frac{1}{u\ln u}\,du = \ln|\ln u| + C$

FORMAS HIPERBÓLICAS

103. $\int \operatorname{senh} u\,du = \cosh u + C$

104. $\int \cosh u\,du = \operatorname{senh} u + C$

105. $\int \tanh u\,du = \ln\cosh u + C$

106. $\int \coth u\,du = \ln|\operatorname{senh} u| + C$

107. $\int \operatorname{sech} u\,du = \tan^{-1}|\operatorname{senh} u| + C$

108. $\int \operatorname{csch} u\,du = \ln|\tanh \tfrac{1}{2}u| + C$

109. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

110. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

111. $\int \operatorname{sech} u\tanh u\,du = -\operatorname{sech} u + C$

112. $\int \operatorname{csch} u\coth u\,du = -\operatorname{csch} u + C$

FORMAS QUE INVOLUCRAN $\sqrt{2au-u^2}$, $a>0$

113. $\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

114. $\int u\sqrt{2au-u^2}\,du = \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

115. $\int \frac{\sqrt{2au-u^2}}{u}\,du = \sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$

116. $\int \frac{\sqrt{2au-u^2}}{u^2}\,du = -\frac{2\sqrt{2au-u^2}}{u} - \cos^{-1}\left(\frac{a-u}{a}\right) + C$

117. $\int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$

118. $\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$

119. $\int \frac{u^2\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

120. $\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$